

Production of Charmed Tetraquarks from B_c and B decays

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Hadronic states composed of multi-quark flavors may exist in reality since they are not prohibited by QCD. Compact four quark systems of color singlet are classified as tetraquarks. To understand the properties of these states, more theoretical and experimental efforts are needed. In this work, we study charmed tetraquarks with three light flavors using flavor $SU(3)$ symmetry. States with three different light quarks must be in a $\mathbf{6}$ or a $\mathbf{15}$ multiplet. We investigate the production of charmed tetraquarks X_c in $B \rightarrow X_c(\bar{X}_c)P$ and $B_c \rightarrow X_cP$ decays. Whether the states with three light quarks belong to $\bar{\mathbf{6}}$ or $\mathbf{15}$ can be determined by studying various tetraquark B and B_c decays. We demonstrate that the decay amplitudes for these decays can be parametrized by a few irreducible $SU(3)$ invariant amplitudes. We then derive relations for decay widths and CP violating rate difference which can be examined experimentally. Although no experimental measurement is available yet, they might be accessed at the ongoing and forthcoming experiments like the LHCb and Belle-II. Measurements of these observables can not only provide useful information for the study of exotics spectroscopy but are also valuable information towards a better understanding of some non-perturbative aspects of QCD.

I. INTRODUCTION

Although most hadrons observed in experiments can be well accommodated in quark model where mesons are composed of a quark and an anti-quark and baryons are made of three quarks, it is widely believed that there exist structures beyond naive quark model. They are generally called as hadron exotics. In the past decades, great progresses have been made in finding exotic hadron states. A milestone is the discovery of $X(3872)$, firstly in B decays by Belle [1] and subsequently confirmed in many reactions by different experiments [2–4]. Since then the identification of the exotic hadrons becomes a key topic in hadron physics. A number of new interesting structures were discovered in the mass region of heavy quarkonium, now generically under the name XYZ states. For recent reviews, see Refs. [5–8].

Many of the newly discovered XYZ states defy the quark model assignment, meson as $\bar{q}q$ or baryon as qqq . Two natural interpretations in theory include the hadron molecule in which XYZ states are formed by two loosely bounded constituents, and the compact tetraquark. However the determination of their internal degrees of freedom may be vague because their quantum numbers can overlap with ordinary mesons and baryons. Thus looking for the four-quark states with different flavors are of unambiguously importance. The D0 collaboration has announced the discovery of a bottomed state $X(5568)$ decaying into the $B_s\pi^\pm$ [9]. Unfortunately, soon after the D0 announcement, the LHCb collaboration reported negative results in their search [10]. The existence of the $X(5568)$ claimed by D0 was not supported by the LHCb data. Nevertheless, the possibility of such exotic state has attracted a lot of theoretical attentions [11–42]. The properties of such exotic states are still being actively studied.

In this work, we propose to search for tetraquarks composed of a charm quark and three different light quarks states, $\bar{c}ds\bar{u}$, $\bar{c}sud$, and $\bar{c}ud\bar{s}$, in B_c and B decays. Whether such states exist will be subject to the future experimental investigations. We first use flavor $SU(3)$ symmetry to classify the charmed tetraquarks X_c , and then estimate their masses by making use of a constituent quark model. Finally we explore B_c and B decay into a X_c state and a pseudoscalar octet state P from flavor $SU(3)$ symmetry respectively. In particular we derive relations for decay widths and CP violations among different decay channels, which can be tested by experiments.

This paper is organized as follows. In Sec. II, we give an overview of $SU(3)$ classification of charmed tetraquarks with different light quarks and their associated members, and estimate their masses. In Sec. III we study the $B_c(B) \rightarrow X_cP$ decay amplitudes using $SU(3)$ symmetry. In Sec. IV we discuss some useful relations for decay widths and CP

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violations in $B_c(B) \rightarrow X_c P$ decays. In the last section, we provide a brief summary of this work.

II. CHARMED TETRAQUARKS SPECTROSCOPY

A. Charmed tetraquarks X_c in $SU(3)$

Charmed tetraquark states $X_c \sim [qq']\bar{q}''\bar{c}$ with three light quarks, q , q' and q'' , can be conveniently organized by flavor $SU(3)$ symmetry [24]. Under the flavor $SU(3)$ symmetry, the three light quarks, (u, d, s) form a triplet **3** representation and the charm quark c is a singlet [43–45]. Tetraquark states formed by three light quarks (q , q' and q'' are different ones) and one charm quark can have the following irreducible representations

$$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15}. \quad (1)$$

We will be interested in charmed tetraquark states with four different quarks, namely $\bar{c}ds\bar{u}$, $\bar{c}su\bar{d}$, and $\bar{c}ud\bar{s}$ states. They must be in a $\bar{\mathbf{6}}$ or a $\mathbf{15}$ representation. We label $\bar{\mathbf{6}}$ representation by $X_{[i,j]}^k$. Here the flavor components are antisymmetric under the exchange of i and j , and traceless $X_{[i,j]}^i = 0$. More explicitly, the components are given by [24]

$$\begin{aligned} X_{[2,3]}^1 &= \frac{1}{\sqrt{2}}X'_{ds\bar{u}}, & X_{[3,1]}^2 &= \frac{1}{\sqrt{2}}X'_{sud}, & X_{[1,2]}^3 &= \frac{1}{\sqrt{2}}X'_{ud\bar{s}}, \\ X_{[1,2]}^1 &= X_{[2,3]}^3 = \frac{1}{2}Y'_{(u\bar{u},s\bar{s})d}, & X_{[3,1]}^1 &= X_{[2,3]}^2 = \frac{1}{2}Y'_{(u\bar{u},d\bar{d})s}, & X_{[1,2]}^2 &= X_{[3,1]}^3 = \frac{1}{2}Y'_{(d\bar{d},s\bar{s})u}. \end{aligned} \quad (2)$$

The $\mathbf{15}$ representation is denoted as $X_{\{i,j\}}^k$. Here the representation is symmetric when exchanging i and j , and traceless $X_{\{i,j\}}^i = 0$ with the components [24]:

$$\begin{aligned} X_{\{2,3\}}^1 &= \frac{1}{\sqrt{2}}X_{ds\bar{u}}, & X_{\{3,1\}}^2 &= \frac{1}{\sqrt{2}}X_{sud}, & X_{\{1,2\}}^3 &= \frac{1}{\sqrt{2}}X_{ud\bar{s}}, \\ X_{\{1,1\}}^1 &= \left(\frac{Y_{\pi u}}{\sqrt{2}} + \frac{Y_{\eta u}}{\sqrt{6}}\right), & X_{\{1,2\}}^1 &= \frac{1}{\sqrt{2}}\left(\frac{Y_{\pi d}}{\sqrt{2}} + \frac{Y_{\eta d}}{\sqrt{6}}\right), & X_{\{1,3\}}^1 &= \frac{1}{\sqrt{2}}\left(\frac{Y_{\pi s}}{\sqrt{2}} + \frac{Y_{\eta s}}{\sqrt{6}}\right), \\ X_{\{2,1\}}^2 &= \frac{1}{\sqrt{2}}\left(-\frac{Y_{\pi u}}{\sqrt{2}} + \frac{Y_{\eta u}}{\sqrt{6}}\right), & X_{\{2,2\}}^2 &= \left(-\frac{Y_{\pi d}}{\sqrt{2}} + \frac{Y_{\eta d}}{\sqrt{6}}\right), & X_{\{2,3\}}^2 &= \frac{1}{\sqrt{2}}\left(-\frac{Y_{\pi s}}{\sqrt{2}} + \frac{Y_{\eta s}}{\sqrt{6}}\right), \\ X_{\{3,1\}}^3 &= -\frac{Y_{\eta u}}{\sqrt{3}}, & X_{\{3,2\}}^3 &= -\frac{Y_{\eta d}}{\sqrt{3}}, & X_{\{3,3\}}^3 &= -\frac{Y_{\eta s}}{\sqrt{3}}, \\ X_{\{2,2\}}^1 &= Z_{dd\bar{u}}, & X_{\{3,3\}}^1 &= Z_{ss\bar{u}}, \\ X_{\{1,1\}}^2 &= Z_{uu\bar{d}}, & X_{\{3,3\}}^2 &= Z_{ss\bar{d}}, \\ X_{\{1,1\}}^3 &= Z_{uu\bar{s}}, & X_{\{2,2\}}^3 &= Z_{dd\bar{s}}. \end{aligned} \quad (3)$$

It is clear that if $SU(3)$ flavor symmetry plays an important role in classifying charmed tetraquark states, there are associated members with those tetraquarks with three different light quarks. Whether they come as a $\bar{\mathbf{6}}$ or $\mathbf{15}$ has to be determined experimentally.

B. Estimation of X_c masses

Before discussing $B_c(B) \rightarrow X_c P$, we estimate the masses for X_c . Here we focus on the lowest-lying tetraquarks where their orbital angular momenta are zero. We assume that the X_c mass is from the constituent quark masses and also various spin-spin correlations as proposed in Ref. [46, 47]. This approach has been applied to various multiquark systems [31, 48–54]. The effective Hamiltonian is given by

$$H = m_\delta + m_{q''} + m_c + H_{SS}^\delta + H_{SS}^{\bar{q}''\bar{c}} + H_{SS}^{\delta\bar{q}''} + H_{SS}^{\delta\bar{c}}, \quad (4)$$

with the spin-spin interactions

$$\begin{aligned}
H_{SS}^\delta &= 2(\kappa_{qq'})_{\bar{3}}(\mathbf{S}_q \cdot \mathbf{S}_{q'}), \\
H_{SS}^{\bar{q}''\bar{c}} &= 2(\kappa_{cq''})_{\bar{3}}(\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}''}), \\
H_{SS}^{\delta\bar{q}''} &= 2\kappa_{q\bar{q}''}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}''}) + 2\kappa_{q'\bar{q}''}(\mathbf{S}_{q'} \cdot \mathbf{S}_{\bar{q}''}), \\
H_{SS}^{\delta\bar{c}} &= 2\kappa_{q\bar{c}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{c}}) + 2\kappa_{q'\bar{c}}(\mathbf{S}_{q'} \cdot \mathbf{S}_{\bar{c}}).
\end{aligned} \tag{5}$$

In the above, the m_δ is the constituent mass of the two quarks $[qq']$ to form a diquark δ . The spin operator of light quarks and heavy antiquark is $\mathbf{S}_{q^{(\prime)}}$ and $\mathbf{S}_{\bar{c}}$, respectively. The spin-spin interaction inside the diquark is denoted as H_{SS}^δ , while the $H_{SS}^{\bar{q}''\bar{c}}$ is the spin-spin interaction between two antiquarks. The $H_{SS}^{\delta\bar{q}''}$ and $H_{SS}^{\delta\bar{c}}$ reflect the spin-spin interaction between the quark and antiquark. The orbital-related terms are neglected for S-wave tetraquark states. The coefficients $\kappa_{q_1\bar{q}_2}$ and $(\kappa_{q_1q'_2})_{\bar{3}}$ correspond to the spin-spin coupling strengths.

The wave function of a tetraquark consists of four parts, i.e. space-coordinate, color, flavor, and spin subspaces:

$$\Psi(q, q', \bar{q}'', \bar{c}) = \psi(x_1, x_2, x_3, x_4) \otimes \chi_c(c_1, c_2, c_3, c_4) \otimes \chi_f(f_1, f_2, f_3, f_4) \otimes \chi_s(s_1, s_2, s_3, s_4), \tag{6}$$

where we use the labels 1, 2, 3, 4 to denote $q, q', \bar{q}'', \bar{c}$, respectively; $\psi(x_i)$, $\chi_c(c_i)$, $\chi_f(f_i)$, and $\chi_s(s_i)$ denote the space, color, flavor, and spin wave functions, respectively. Since we focus on the tetraquarks with $L = 0$, the space wave function is symmetric. The diquark is attractive only in the triplet representation in color space, thus the color wave function is antisymmetric.

If the spin wave function of the light quark system $[qq']$ is antisymmetric, i.e. $S_\delta = 0$, the flavor function should be also antisymmetric. In this case, the charmed tetraquarks can be decomposed into the **6** representation, with the spin-parity $J^P = 0^+, 1^+$:

$$\begin{aligned}
|0_\delta, \frac{1}{2}_{\bar{q}''}; \frac{1}{2}_{\delta\bar{q}''}, \frac{1}{2}_{\bar{c}}, 0_J\rangle &= \frac{1}{2}[(\uparrow)_q(\downarrow)_{q'} - (\downarrow)_q(\uparrow)_{q'}][(\uparrow)_{\bar{q}''}(\downarrow)_{\bar{c}} - (\downarrow)_{\bar{q}''}(\uparrow)_{\bar{c}}], \\
|0_\delta, \frac{1}{2}_{\bar{q}''}; \frac{1}{2}_{\delta\bar{q}''}, \frac{1}{2}_{\bar{c}}, 1_J\rangle &= \frac{1}{\sqrt{2}}[(\uparrow)_q(\downarrow)_{q'} - (\downarrow)_q(\uparrow)_{q'}](\uparrow)_{\bar{q}''}(\uparrow)_{\bar{c}},
\end{aligned} \tag{7}$$

In the above, the tetraquark states are classified according to $|S_\delta, S_{\bar{q}''}; S_{\delta\bar{q}''}, S_{\bar{c}}, S_J\rangle$; the $S_\delta, S_{\bar{q}''}, S_{\bar{c}}$ and $S_{\delta\bar{q}''}$ stand for the spins of diquark $[qq']$, antiquark, heavy antiquark, and $[qq']\bar{q}''$, respectively, while the S_J is the total angular momentum.

Using the basis defined in Eq. (7), one can derive the mass matrix for the $J^P = 0^+$ and $J^P = 1^+$ tetraquarks in the **6** representation

$$\begin{aligned}
M(0^+) &= m_\delta + m_{q''} + m_c - \frac{3}{2}((\kappa_{qq'})_{\bar{3}} + (\kappa_{cq''})_{\bar{3}}), \\
M(1^+) &= m_\delta + m_{q''} + m_c - \frac{1}{2}(3(\kappa_{qq'})_{\bar{3}} - (\kappa_{cq''})_{\bar{3}}),
\end{aligned} \tag{8}$$

where we use the flavor SU(3) symmetry $\kappa_{q\bar{c}} = \kappa_{q'\bar{c}}$ and $\kappa_{q\bar{q}''} = \kappa_{q'\bar{q}''}$ for simplification, and this leads to the vanishing contribution from both $H_{SS}^{\delta\bar{q}''}$ and $H_{SS}^{\delta\bar{c}}$ interactions, because $\mathbf{S}_q + \mathbf{S}_{q'} = \mathbf{S}_\delta = 0$.

If the spin wave function of the light quark system $[qq']$ is symmetric, namely $S_\delta = 1$, the flavor function is also symmetric. In this case, the charmed tetraquarks can be decomposed into the **15** representation. The spin-parity could be $J^P = 0^+, 1^+, 2^+$:

$$\begin{aligned}
|1_\delta, \frac{1}{2}_{\bar{q}''}; \frac{1}{2}_{\delta\bar{q}''}, \frac{1}{2}_{\bar{c}}, 0_J\rangle &= \frac{1}{\sqrt{3}}\{(\uparrow)_q(\uparrow)_{q'}(\downarrow)_{\bar{q}''}(\downarrow)_{\bar{c}} + (\downarrow)_q(\downarrow)_{q'}(\uparrow)_{\bar{q}''}(\uparrow)_{\bar{c}} - \frac{1}{2}[(\uparrow)_q(\downarrow)_{q'} + (\downarrow)_q(\uparrow)_{q'}][(\uparrow)_{\bar{q}''}(\downarrow)_{\bar{c}} + (\downarrow)_{\bar{q}''}(\uparrow)_{\bar{c}}]\}, \\
|1_\delta, \frac{1}{2}_{\bar{q}''}; \frac{1}{2}_{\delta\bar{q}''}, \frac{1}{2}_{\bar{c}}, 1_J\rangle &= \frac{1}{\sqrt{6}}\{2(\uparrow)_q(\uparrow)_{q'}(\downarrow)_{\bar{q}''}(\uparrow)_{\bar{c}} - [(\uparrow)_q(\downarrow)_{q'} + (\downarrow)_q(\uparrow)_{q'}](\uparrow)_{\bar{q}''}(\uparrow)_{\bar{c}}\}, \\
|1_\delta, \frac{1}{2}_{\bar{q}''}; \frac{3}{2}_{\delta\bar{q}''}, \frac{1}{2}_{\bar{c}}, 1_J\rangle &= \frac{1}{2\sqrt{3}}\{3(\uparrow)_q(\uparrow)_{q'}(\uparrow)_{\bar{q}''}(\downarrow)_{\bar{c}} - (\uparrow)_q(\uparrow)_{q'}(\downarrow)_{\bar{q}''}(\uparrow)_{\bar{c}} - [(\uparrow)_q(\downarrow)_{q'} + (\downarrow)_q(\uparrow)_{q'}](\uparrow)_{\bar{q}''}(\uparrow)_{\bar{c}}\}, \\
|1_\delta, \frac{1}{2}_{\bar{q}''}; \frac{3}{2}_{\delta\bar{q}''}, \frac{1}{2}_{\bar{c}}, 2_J\rangle &= (\uparrow)_q(\uparrow)_{q'}(\uparrow)_{\bar{q}''}(\uparrow)_{\bar{c}}.
\end{aligned} \tag{9}$$

The masses for the $J^P = 0^+$ and $J^P = 2^+$ tetraquarks in the **15** representation are given as

$$\begin{aligned}
M(0^+) &= m_\delta + m_{q''} + m_c + \frac{1}{2}((\kappa_{qq'})_{\bar{3}} + (\kappa_{cq''})_{\bar{3}}) - 2(\kappa_{q\bar{q}''} + \kappa_{q\bar{c}}), \\
M(2^+) &= m_\delta + m_{q''} + m_c + \kappa_{q\bar{q}''} + \kappa_{q\bar{c}} + \frac{1}{2}((\kappa_{qq'})_{\bar{3}} + (\kappa_{cq''})_{\bar{3}}).
\end{aligned} \tag{10}$$

Note that there are two possible ways for the charmed tetraquark with spin-parity $J^P = 1^+$. One of them is from the light quark system $qq'\bar{q}''$ having the spin $\frac{1}{2}$ and combining to the total spin 1 with the heavy antiquark, while the other one is from the light quark system $qq'\bar{q}''$ having the spin $\frac{3}{2}$ and combining to the total spin 1 with the heavy antiquark. They mix with each other, since they have the same quantum numbers. Using the second and third basis defined in Eq. (9), one can obtain the mass matrix M for $J^P = 1^+$ tetraquarks in the **15** representation

$$M(1^+) = m_\delta + m_{q''} + m_c + \frac{1}{2}(\kappa_{qq'})_{\bar{3}} + \begin{pmatrix} -2\kappa_{q\bar{q}'} + \frac{2}{3}\kappa_{q\bar{c}} - \frac{1}{6}(\kappa_{q''c})_{\bar{3}} & \frac{2}{3}\sqrt{2}((\kappa_{q''c})_{\bar{3}} - \kappa_{q\bar{c}}) \\ \frac{2}{3}\sqrt{2}((\kappa_{q''c})_{\bar{3}} - \kappa_{q\bar{c}}) & \kappa_{q\bar{q}'} - \frac{5}{6}(2\kappa_{q\bar{c}} + (\kappa_{q''c})_{\bar{3}}) \end{pmatrix}. \quad (11)$$

Diagonalizing the above matrix, one obtains two different eigenvalues of masses.

In the flavor $SU(3)$ symmetry, all charmed tetraquark states will have the identical masses. By distinguishing the strange quark from the up and down quarks, one can obtain the charmed tetraquark masses including the $SU(3)$ symmetry breaking effects. In the numerical calculation, we will use the quark masses as $m_q = 305\text{MeV}$, $m_s = 490\text{MeV}$, $m_c = 1.670\text{GeV}$ [46, 53]. For the light diquark $\delta = [qq]$, we use $m_{qq} = 0.395\text{GeV}$ and $m_{sq} = 0.590\text{GeV}$ [46]. The strange diquark mass $m_{ss} = 0.785\text{GeV}$ is estimated by the relation $m_{ss} - m_{sq} = m_{sq} - m_{qq}$. The spin-spin couplings are $(\kappa_{qq})_{\bar{3}} = 103\text{MeV}$, $(\kappa_{sq})_{\bar{3}} = 64\text{MeV}$, $(\kappa_{cq})_{\bar{3}} = 22\text{MeV}$, $(\kappa_{cs})_{\bar{3}} = 25\text{MeV}$, $(\kappa_{ss})_{\bar{3}} = 72\text{MeV}$, $(\kappa_{q\bar{q}})_0 = 315\text{MeV}$, $(\kappa_{s\bar{q}})_0 = 195\text{MeV}$, $(\kappa_{s\bar{s}})_0 = 121\text{MeV}$, $(\kappa_{c\bar{q}})_0 = 70\text{MeV}$ and $(\kappa_{c\bar{s}})_0 = 72\text{MeV}$ [46, 53]. The relation $\kappa_{ij} = \frac{1}{4}(\kappa_{ij})_0$ for the quark-antiquark state derived from one gluon exchange model has been employed.

For the tetraquarks in the $\bar{\mathbf{6}}$ representation, their masses are estimated to be:

$$m(X'_{ds\bar{u}}) = m(X'_{su\bar{d}}) = m(Y'_{(u\bar{u},d\bar{d})s}) = \begin{cases} 2.44\text{GeV}, & J^P = 0^+, \\ 2.48\text{GeV}, & J^P = 1^+, \end{cases} \quad (12)$$

$$m(X'_{ud\bar{s}}) = \begin{cases} 2.36\text{GeV}, & J^P = 0^+, \\ 2.41\text{GeV}, & J^P = 1^+, \end{cases} \quad (13)$$

$$m(Y'_{(u\bar{u},s\bar{s})d}) = m(Y'_{(d\bar{d},s\bar{s})u}) = \begin{cases} 2.40\text{GeV}, & J^P = 0^+, \\ 2.45\text{GeV}, & J^P = 1^+. \end{cases} \quad (14)$$

The spin of charmed tetraquark states in the **15** representation could be 0, 1, and 2. We give the predictions for their masses:

$$m(X_{ds\bar{u}}) = m(X_{su\bar{d}}) = m(Y_{\pi s}) = \begin{cases} 2.47\text{GeV}, & J^P = 0^+, \\ 2.51\text{GeV}, 2.60\text{GeV}, & J^P = 1^+, \\ 2.67\text{GeV}, & J^P = 2^+, \end{cases} \quad (15)$$

$$m(X_{ud\bar{s}}) = m(Z_{uu\bar{s}}) = m(Z_{dd\bar{s}}) = \begin{cases} 2.49\text{GeV}, & J^P = 0^+, \\ 2.52\text{GeV}, 2.61\text{GeV}, & J^P = 1^+, \\ 2.69\text{GeV}, & J^P = 2^+, \end{cases} \quad (16)$$

$$m(Y_{\pi u}) = m(Y_{\pi d}) = m(Z_{uu\bar{d}}) = m(Z_{dd\bar{u}}) = \begin{cases} 2.24\text{GeV}, & J^P = 0^+, \\ 2.27\text{GeV}, 2.45\text{GeV}, & J^P = 1^+, \\ 2.53\text{GeV}, & J^P = 2^+, \end{cases} \quad (17)$$

$$m(Y_{\eta u}) = m(Y_{\eta d}) = \begin{cases} 2.55\text{GeV}, & J^P = 0^+, \\ 2.58\text{GeV}, 2.66\text{GeV}, & J^P = 1^+, \\ 2.74\text{GeV}, & J^P = 2^+, \end{cases} \quad (18)$$

$$m(Y_{\eta s}) = \begin{cases} 2.76\text{GeV}, & J^P = 0^+, \\ 2.79\text{GeV}, 2.84\text{GeV}, & J^P = 1^+, \\ 2.92\text{GeV}, & J^P = 2^+, \end{cases} \quad (19)$$

$$m(Z_{ss\bar{u}}) = m(Z_{ss\bar{d}}) = \begin{cases} 2.71\text{GeV}, & J^P = 0^+, \\ 2.74\text{GeV}, 2.78\text{GeV}, & J^P = 1^+, \\ 2.86\text{GeV}, & J^P = 2^+. \end{cases} \quad (20)$$

The masses of X_c are in the range between 2.24GeV and 2.92GeV. The above discussion shows that there should be enough phase space to allow $B_c(B) \rightarrow X_c P$ occur.

III. EFFECTIVE HAMILTONIAN AND DECAY AMPLITUDES FOR $B_c(B) \rightarrow X_c P$

In this section, we study the $B_c \rightarrow X_c P$ and $B \rightarrow X_c P$ decays. Let us first identify the flavor $SU(3)$ symmetry properties of the particles involved. As already mentioned before X_c can be in $\bar{\mathbf{6}}$ or **15**. The other $SU(3)$ transformation

TABLE I: Decay amplitudes of $B_c \rightarrow X_c P$ decays into a $\bar{\mathbf{6}}$ charmed tetraquark.

channel $\Delta S = 0$	amplitude	channel $\Delta S = 1$	amplitude
$B_c^- \rightarrow K^- X'_{ud\bar{s}}$	$-\frac{a_3 - 2a_6 - a_{15} + b_6}{\sqrt{2}}$	$B_c^- \rightarrow \pi^- X'_{su\bar{d}}$	$\frac{a_3 - 2a_6 - a_{15} + b_6}{\sqrt{2}}$
$B_c^- \rightarrow \pi^- Y'_{(d\bar{d}, s\bar{s})u}$	$-\frac{a_3 - 2a_6 - a_{15} + b_6}{2}$	$B_c^- \rightarrow K^- Y'_{(d\bar{d}, s\bar{s})u}$	$\frac{a_3 - 2a_6 - a_{15} + b_6}{2}$
$B_c^- \rightarrow K^+ X'_{ds\bar{u}}$	$\frac{a_3 + 2a_6 + 3a_{15} - b_6}{\sqrt{2}}$	$B_c^- \rightarrow \pi^+ X'_{ds\bar{u}}$	$-\frac{a_3 + 2a_6 + 3a_{15} - b_6}{\sqrt{2}}$
$B_c^- \rightarrow K^0 Y'_{(u\bar{u}, d\bar{d})s}$	$\frac{a_3 + 2a_6 - 5a_{15} - b_6}{2}$	$B_c^- \rightarrow \bar{K}^0 Y'_{(u\bar{u}, s\bar{s})d}$	$-\frac{a_3 + 2a_6 - 5a_{15} - b_6}{2}$
$B_c^- \rightarrow \pi^0 Y'_{(u\bar{u}, s\bar{s})d}$	$-\frac{a_3 - 2a_6 + 7a_{15} + b_6}{2\sqrt{2}}$	$B_c^- \rightarrow \pi^0 Y'_{(u\bar{u}, d\bar{d})s}$	$\frac{a_3 + a_{15}}{\sqrt{2}}$
$B_c^- \rightarrow \eta Y'_{(u\bar{u}, s\bar{s})d}$	$-\frac{3a_3 + 2a_6 - 3a_{15} - b_6}{2\sqrt{6}}$	$B_c^- \rightarrow \eta Y'_{(u\bar{u}, d\bar{d})s}$	$-\frac{2a_6 - 6a_{15} - b_6}{\sqrt{6}}$

properties are: the B_c is a singlet in the $SU(3)$, while the $B_i = (B_u(u\bar{b}), B_d(d\bar{b}), B_s(s\bar{b}))$ transform as $\mathbf{3}$ representation, and the pseudo-scalar meson P is an octet:

$$P_j^i = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta}{\sqrt{6}} \end{pmatrix}. \quad (21)$$

A. $B_c \rightarrow X_c P$ decays

The $B_c \rightarrow X_c P$ decays are induced by charmless $b \rightarrow q$ ($q = d, s$) transition. The weak Hamiltonian \mathcal{H}_{eff} is given by:

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{uq}^* [C_1 O_1^{\bar{u}u} + C_2 O_2^{\bar{u}u}] - V_{tb} V_{tq}^* \left[\sum_{i=3}^{10} C_i O_i \right] \right\} + \text{H.c.}, \quad (22)$$

where the V_{ij} is CKM matrix element. The O_i is a four-quark operator or a moment type operator. At the hadron level, penguin operators behave as the $\mathbf{3}$ representation while tree operators in Eq. (22) transform under the flavor $SU(3)$ symmetry as $\bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} = \bar{\mathbf{3}} \oplus \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \mathbf{15}$. So the Hamiltonian can be decomposed in terms of a vector $H^i(\bar{\mathbf{3}})$, a traceless tensor antisymmetric in upper indices, $H_k^{[ij]}(\mathbf{6})$, and a traceless tensor symmetric in upper indices, $H_k^{\{ij\}}(\mathbf{15})$.

For the $\Delta S = 0(b \rightarrow d)$ decays, the non-zero components of the effective Hamiltonian are [55–57]:

$$\begin{aligned} H^2(\bar{\mathbf{3}}) &= 1, \quad H_1^{12}(\mathbf{6}) = -H_1^{21}(\mathbf{6}) = H_3^{23}(\mathbf{6}) = -H_3^{32}(\mathbf{6}) = 1, \\ 2H_1^{12}(\mathbf{15}) &= 2H_1^{21}(\mathbf{15}) = -3H_2^{22}(\mathbf{15}) = -6H_3^{23}(\mathbf{15}) = -6H_3^{32}(\mathbf{15}) = 6, \end{aligned} \quad (23)$$

with all other remaining entries zero. For the $\Delta S = 1(b \rightarrow s)$ decays the nonzero entries in $H^i(\bar{\mathbf{3}})$, $H_k^{[ij]}(\mathbf{6})$, $H_k^{\{ij\}}(\mathbf{15})$ are obtained from Eq. (23) with the exchange $2 \leftrightarrow 3$.

The decay amplitudes $A(B_c \rightarrow X_c P) = \langle X_c P | \mathcal{H}_{eff} | B_c \rangle$ can be separated into two parts, tree and the penguin amplitudes $A_{B_c}^T$ and $A_{B_c}^P$, with

$$A(B_c \rightarrow X_c P) = V_{ub} V_{uq}^* A_{B_c}^T + V_{tb} V_{tq}^* A_{B_c}^P. \quad (24)$$

The $A_{B_c}^{\alpha=T,P}$ are then obtained by contracting the $SU(3)$ indices in all possible ways. Each independent way will have an irreducible and non-perturbative $SU(3)$ amplitude. For the $\bar{\mathbf{6}}$ charmed tetraquark, we have

$$A_{B_c}^\alpha = a_3^\alpha H^i(\bar{\mathbf{3}}) X_{[i,j]}^k P_k^j + a_6^\alpha H_l^{[ij]}(\mathbf{6}) X_{[i,j]}^k P_k^l + b_6^\alpha H_k^{[il]}(\mathbf{6}) X_{[i,j]}^k P_l^j + a_{15}^\alpha H_k^{\{il\}}(\mathbf{15}) X_{[i,j]}^k P_l^j. \quad (25)$$

For the $\mathbf{15}$ charmed tetraquark, we similarly have

$$A_{B_c}^\alpha = c_3^\alpha H^i(\bar{\mathbf{3}}) X_{\{i,j\}}^k P_k^j + c_{15}^\alpha H_l^{\{ij\}}(\mathbf{15}) X_{\{i,j\}}^k P_k^l + d_{15}^\alpha H_k^{\{il\}}(\mathbf{15}) X_{\{i,j\}}^k P_l^j + c_6^\alpha H_k^{[il]}(\mathbf{6}) X_{\{i,j\}}^k P_l^j. \quad (26)$$

Expanding the above expressions, one can obtain the decay amplitudes $A_{B_c}^\alpha$. Results for the $A_{B_c}^T$ are collected in Tables I, II. In these tables, we have dropped the superscript $\alpha = T$ for simplicity. Penguin amplitudes $A_{B_c}^P$ have similar expressions but only the triplet contributions containing α_3^P are nonzero.

TABLE II: Decay amplitudes of $B_c \rightarrow X_c P$ decays into a tetraquark in the **15** representation.

channel $\Delta S = 0$	amplitude	channel $\Delta S = 1$	amplitude
$B_c^- \rightarrow K^- X_{ud\bar{s}}$	$\frac{c_3+c_6+6c_{15}-d_{15}}{\sqrt{2}}$	$B_c^- \rightarrow \pi^- X_{su\bar{d}}$	$\frac{c_3+c_6+6c_{15}-d_{15}}{\sqrt{2}}$
$B_c^- \rightarrow K^+ X_{ds\bar{u}}$	$\frac{c_3-c_6-2c_{15}+3d_{15}}{\sqrt{2}}$	$B_c^- \rightarrow \pi^+ X_{ds\bar{u}}$	$\frac{c_3-c_6-2c_{15}+3d_{15}}{\sqrt{2}}$
$B_c^- \rightarrow \pi^+ Z_{dd\bar{u}}$	$c_3 - c_6 - 2c_{15} + 3d_{15}$	$B_c^- \rightarrow K^+ Z_{ss\bar{u}}$	$c_3 - c_6 - 2c_{15} + 3d_{15}$
$B_c^- \rightarrow \bar{K}^0 Z_{dd\bar{s}}$	$c_3 + c_6 - 2c_{15} - d_{15}$	$B_c^- \rightarrow K^0 Z_{ss\bar{d}}$	$c_3 + c_6 - 2c_{15} - d_{15}$
$B_c^- \rightarrow \pi^0 Y_{\pi d}$	$\frac{(2+\sqrt{2})c_3-2\sqrt{2}c_6-2(2-3\sqrt{2})c_{15}-4d_{15}}{4}$	$B_c^- \rightarrow \pi^0 Y_{\pi s}$	$\frac{c_3+2c_{15}+d_{15}}{\sqrt{2}}$
$B_c^- \rightarrow \pi^0 Y_{\eta d}$	$\frac{(\sqrt{2}-2)c_3-4\sqrt{2}c_6+2(3\sqrt{2}+2)c_{15}-2(\sqrt{2}-2)d_{15}}{4\sqrt{3}}$	$B_c^- \rightarrow \pi^0 Y_{\eta s}$	$\frac{-c_6+4c_{15}+2d_{15}}{\sqrt{6}}$
$B_c^- \rightarrow \pi^- Y_{\pi u}$	$-\frac{c_3}{2} + \frac{c_6}{\sqrt{2}} - 3c_{15} + \frac{3d_{15}}{\sqrt{2}} + d_{15}$	$B_c^- \rightarrow K^- Y_{\pi u}$	$\frac{(1+\sqrt{2})c_6+(1+3\sqrt{2})d_{15}}{2}$
$B_c^- \rightarrow \pi^- Y_{\eta u}$	$\frac{c_3+(2+\sqrt{2})c_6+6c_{15}+3\sqrt{2}d_{15}}{2\sqrt{3}}$	$B_c^- \rightarrow \bar{K}^0 Y_{\pi d}$	$\frac{(1+\sqrt{2})c_6+(3+\sqrt{2})d_{15}}{2}$
$B_c^- \rightarrow K^0 Y_{\pi s}$	$-\frac{(c_3-c_6-2c_{15}-5d_{15})}{2}$	$B_c^- \rightarrow \bar{K}^0 Y_{\eta d}$	$-\frac{2c_3+(\sqrt{2}-1)c_6-4c_{15}-(7-\sqrt{2})d_{15}}{2\sqrt{3}}$
$B_c^- \rightarrow K^0 Y_{\eta s}$	$\frac{c_3+3c_6-2c_{15}+3d_{15}}{2\sqrt{3}}$	$B_c^- \rightarrow K^- Y_{\eta u}$	$-\frac{2c_3-(\sqrt{2}-1)c_6-3(-4c_{15}+(1+\sqrt{2})d_{15})}{2\sqrt{3}}$
$B_c^- \rightarrow \eta Y_{\pi d}$	$\frac{(\sqrt{2}-2)c_3+2(2+3\sqrt{2})(c_{15}+d_{15})}{4\sqrt{3}}$	$B_c^- \rightarrow \eta Y_{\pi s}$	$-\frac{3c_6-4c_{15}+2d_{15}}{\sqrt{6}}$
$B_c^- \rightarrow \eta Y_{\eta d}$	$\frac{(2+5\sqrt{2})c_3+2(3\sqrt{2}c_6-(2+\sqrt{2})c_{15}+2(\sqrt{2}-1)d_{15})}{12}$	$B_c^- \rightarrow \eta Y_{\eta s}$	$\frac{3c_3-2c_{15}-5d_{15}}{3\sqrt{2}}$

TABLE III: Decay amplitudes of $B \rightarrow \bar{X}_c P$ decays into a charmed tetraquark in the **6** representation.

channel $\Delta S = 0$	amplitude	channel $\Delta S = 1$	amplitude
$B^- \rightarrow K^- \bar{X}'_{su\bar{d}}$	$-\frac{b_8+c_8}{\sqrt{2}}$	$B^- \rightarrow \pi^- \bar{X}'_{ud\bar{s}}$	$\frac{b_8+c_8}{\sqrt{2}}$
$B^- \rightarrow \pi^- \bar{Y}'_{(d\bar{d},s\bar{s})u}$	$\frac{b_8+c_8}{2}$	$B^- \rightarrow K^- \bar{Y}'_{(d\bar{d},s\bar{s})u}$	$-\frac{b_8+c_8}{2}$
$\bar{B}^0 \rightarrow \bar{K}^0 \bar{X}'_{su\bar{d}}$	$-\frac{a_8+b_8}{\sqrt{2}}$	$\bar{B}_s^0 \rightarrow K^0 \bar{X}'_{ud\bar{s}}$	$\frac{a_8+b_8}{\sqrt{2}}$
$\bar{B}^0 \rightarrow \pi^- \bar{Y}'_{(u\bar{u},s\bar{s})d}$	$\frac{a_8-d_8}{2}$	$\bar{B}_s^0 \rightarrow K^- \bar{Y}'_{(u\bar{u},d\bar{d})s}$	$-\frac{a_8-d_8}{2}$
$\bar{B}^0 \rightarrow K^0 \bar{X}'_{ud\bar{s}}$	$\frac{a_8-d_8}{\sqrt{2}}$	$\bar{B}_s^0 \rightarrow \bar{K}^0 \bar{X}'_{su\bar{d}}$	$-\frac{a_8-d_8}{\sqrt{2}}$
$\bar{B}^0 \rightarrow K^- \bar{Y}'_{(u\bar{u},d\bar{d})s}$	$-\frac{a_8-c_8}{2}$	$\bar{B}_s^0 \rightarrow \pi^- \bar{Y}'_{(u\bar{u},s\bar{s})d}$	$\frac{a_8-c_8}{2}$
$\bar{B}^0 \rightarrow \pi^0 \bar{Y}'_{(d\bar{d},s\bar{s})u}$	$-\frac{a_8+b_8+c_8-d_8}{2\sqrt{2}}$	$\bar{B}_s^0 \rightarrow \pi^0 \bar{Y}'_{(d\bar{d},s\bar{s})u}$	$-\frac{a_8-c_8}{2\sqrt{2}}$
$\bar{B}^0 \rightarrow \eta \bar{Y}'_{(d\bar{d},s\bar{s})u}$	$\frac{3a_8+b_8-c_8-d_8}{2\sqrt{6}}$	$\bar{B}_s^0 \rightarrow \eta \bar{Y}'_{(d\bar{d},s\bar{s})u}$	$\frac{3a_8+2b_8+c_8-2d_8}{2\sqrt{6}}$
$\bar{B}_s^0 \rightarrow K^0 \bar{Y}'_{(d\bar{d},s\bar{s})u}$	$\frac{b_8+d_8}{2}$	$\bar{B}^0 \rightarrow \bar{K}^0 \bar{Y}'_{(d\bar{d},s\bar{s})u}$	$-\frac{b_8+d_8}{2}$
$\bar{B}_s^0 \rightarrow \pi^- \bar{Y}'_{(u\bar{u},d\bar{d})s}$	$-\frac{c_8-d_8}{2}$	$\bar{B}^0 \rightarrow K^- \bar{Y}'_{(u\bar{u},s\bar{s})d}$	$\frac{c_8-d_8}{2}$
$\bar{B}_s^0 \rightarrow \pi^0 \bar{X}'_{su\bar{d}}$	$\frac{c_8-d_8}{2}$	$\bar{B}^0 \rightarrow \pi^0 \bar{X}'_{ud\bar{s}}$	$-\frac{b_8+c_8}{2}$
$\bar{B}_s^0 \rightarrow \eta \bar{X}'_{su\bar{d}}$	$\frac{2b_8+c_8+d_8}{2\sqrt{3}}$	$\bar{B}^0 \rightarrow \eta \bar{X}'_{ud\bar{s}}$	$\frac{b_8-c_8+2d_8}{2\sqrt{3}}$

B. $B \rightarrow \bar{X}_c P$ decays

For $B \rightarrow \bar{X}_c(X_c)P$ decays, the b -quark in B should decay into a charm (anti-charm) quark so that X_c or its charge conjugate can be generated. The operator to produce a charm quark from a b -quark, $\bar{c}b\bar{q}u$ is given by

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* [C_1 O_1^{\bar{c}u} + C_2 O_2^{\bar{c}u}] + \text{H.c.} \quad (27)$$

The light quarks in this effective Hamiltonian form an octet with the nonzero entry

$$H_1^2(\mathbf{8}) = 1 \quad (28)$$

for the $b \rightarrow \bar{c}ud$ transition, and $H_1^3(\mathbf{8}) = 1$ for the $b \rightarrow \bar{c}us$ transition.

Similarly as for the B decays, one can write down the irreducible $SU(3)$ amplitudes for $B \rightarrow \bar{X}_c P$ decays. This time, there exist only tree amplitudes which one can normalize $A(B \rightarrow \bar{X}_c P) = V_{cb} V_{uq}^* A_{B\bar{X}_c}$. For **6** charmed tetraquarks, the decay amplitudes can be constructed as

$$A_{B\bar{X}_c} = a_8 B_i H_j^i(\mathbf{8}) \bar{X}_k^{[j,l]} P_l^k + b_8 B_i H_j^k(\mathbf{8}) \bar{X}_k^{[j,l]} P_l^i + c_8 B_j H_i^k(\mathbf{8}) \bar{X}_k^{[j,l]} P_l^i + d_8 B_j H_l^i(\mathbf{8}) \bar{X}_k^{[j,l]} P_i^k. \quad (29)$$

TABLE IV: Decay amplitudes of $B \rightarrow \bar{X}_c P$ decays into the $\bar{\mathbf{15}}$ representation.

channel $\Delta S = 0$	amplitude	channel $\Delta S = 1$	amplitude
$B^- \rightarrow K^- \bar{X}_{sud}$	$\frac{f_8+g_8}{\sqrt{2}}$	$B^- \rightarrow \pi^- \bar{X}_{ud\bar{s}}$	$\frac{f_8+g_8}{\sqrt{2}}$
$B^- \rightarrow \pi^0 \bar{Z}_{uud}$	$\frac{f_8+g_8-h_8}{\sqrt{2}}$	$B^- \rightarrow \pi^0 \bar{Z}_{uus}$	$\frac{f_8+g_8}{\sqrt{2}}$
$B^- \rightarrow K^0 \bar{Z}_{uus}$	h_8	$B^- \rightarrow \bar{K}^0 \bar{Z}_{uud}$	h_8
$B^- \rightarrow \eta \bar{Z}_{uud}$	$\frac{f_8+g_8+h_8}{\sqrt{6}}$	$B^- \rightarrow \eta \bar{Z}_{uus}$	$\frac{f_8+g_8-2h_8}{\sqrt{6}}$
$B^- \rightarrow \pi^- \bar{Y}_{\pi u}$	$\frac{-f_8-g_8+\sqrt{2}h_8}{2}$	$B^- \rightarrow K^- \bar{Y}_{\pi u}$	$\frac{h_8}{\sqrt{2}}$
$B^- \rightarrow \pi^- \bar{Y}_{\eta u}$	$\frac{f_8+g_8+\sqrt{2}h_8}{2\sqrt{3}}$	$B^- \rightarrow K^- \bar{Y}_{\eta u}$	$\frac{-2f_8-2g_8+\sqrt{2}h_8}{2\sqrt{3}}$
$\bar{B}^0 \rightarrow K^0 \bar{X}_{ud\bar{s}}$	$\frac{e_8+h_8}{\sqrt{2}}$	$\bar{B}_s^0 \rightarrow K^0 \bar{X}_{ud\bar{s}}$	$\frac{e_8+f_8}{\sqrt{2}}$
$\bar{B}^0 \rightarrow \bar{K}^0 \bar{X}_{sud}$	$\frac{e_8+f_8}{\sqrt{2}}$	$\bar{B}_s^0 \rightarrow \bar{K}^0 \bar{X}_{sud}$	$\frac{e_8+h_8}{\sqrt{2}}$
$\bar{B}^0 \rightarrow \pi^+ \bar{Z}_{uud}$	$e_8 + f_8$	$\bar{B}_s^0 \rightarrow K^+ \bar{Z}_{uus}$	$e_8 + f_8$
$\bar{B}^0 \rightarrow K^+ \bar{Z}_{uus}$	e_8	$\bar{B}_s^0 \rightarrow \pi^+ \bar{Z}_{uud}$	e_8
$\bar{B}^0 \rightarrow \pi^0 \bar{Y}_{\pi u}$	$\frac{(2+\sqrt{2})e_8+\sqrt{2}(f_8-g_8+h_8)}{4}$	$\bar{B}_s^0 \rightarrow \pi^0 \bar{Y}_{\pi u}$	$\frac{(2+\sqrt{2})e_8}{4}$
$\bar{B}^0 \rightarrow \pi^0 \bar{Y}_{\eta u}$	$-\frac{(\sqrt{2}-2)e_8+\sqrt{2}(f_8-g_8+h_8)}{4\sqrt{3}}$	$\bar{B}_s^0 \rightarrow \pi^0 \bar{Y}_{\eta u}$	$\frac{-\sqrt{3}(\sqrt{2}-2)e_8-2\sqrt{6}g_8}{12}$
$\bar{B}^0 \rightarrow \pi^- \bar{Y}_{\pi d}$	$\frac{e_8-\sqrt{2}g_8+h_8}{2}$	$\bar{B}_s^0 \rightarrow \pi^- \bar{Y}_{\pi d}$	$\frac{e_8}{2}$
$\bar{B}^0 \rightarrow \pi^- \bar{Y}_{\eta d}$	$\frac{e_8+\sqrt{2}g_8+h_8}{2\sqrt{3}}$	$\bar{B}_s^0 \rightarrow \pi^- \bar{Y}_{\eta d}$	$\frac{e_8-2g_8}{2\sqrt{3}}$
$\bar{B}^0 \rightarrow K^- \bar{Y}_{\pi s}$	$\frac{e_8-g_8}{2}$	$\bar{B}_s^0 \rightarrow K^- \bar{Y}_{\pi s}$	$\frac{e_8+h_8}{2}$
$\bar{B}^0 \rightarrow K^- \bar{Y}_{\eta s}$	$\frac{e_8+g_8}{2\sqrt{3}}$	$\bar{B}_s^0 \rightarrow K^- \bar{Y}_{\eta s}$	$\frac{e_8-2g_8+h_8}{2\sqrt{3}}$
$\bar{B}^0 \rightarrow \eta \bar{Y}_{\pi u}$	$-\frac{(\sqrt{2}-2)e_8+\sqrt{2}(f_8+g_8+h_8)}{4\sqrt{3}}$	$\bar{B}_s^0 \rightarrow \eta \bar{Y}_{\pi u}$	$-\frac{(\sqrt{2}-2)e_8}{4\sqrt{3}}$
$\bar{B}^0 \rightarrow \eta \bar{Y}_{\eta u}$	$\frac{(2+5\sqrt{2})e_8+\sqrt{2}(f_8+g_8+h_8)}{12}$	$\bar{B}_s^0 \rightarrow \eta \bar{Y}_{\eta u}$	$\frac{(2+5\sqrt{2})e_8+2\sqrt{2}(2f_8-g_8+2h_8)}{12}$
$\bar{B}_s^0 \rightarrow \pi^0 \bar{X}_{sud}$	$\frac{g_8-h_8}{2}$	$\bar{B}^0 \rightarrow \pi^0 \bar{X}_{ud\bar{s}}$	$\frac{g_8-f_8}{2}$
$\bar{B}_s^0 \rightarrow \eta \bar{X}_{sud}$	$\frac{-2f_8+g_8+h_8}{2\sqrt{3}}$	$\bar{B}^0 \rightarrow \eta \bar{X}_{ud\bar{s}}$	$\frac{f_8+g_8-2h_8}{2\sqrt{3}}$
$\bar{B}_s^0 \rightarrow K^+ \bar{Z}_{uud}$	f_8	$\bar{B}^0 \rightarrow \pi^+ \bar{Z}_{uus}$	f_8
$\bar{B}_s^0 \rightarrow K^- \bar{Z}_{ss\bar{d}}$	g_8	$\bar{B}^0 \rightarrow \pi^- \bar{Z}_{dd\bar{s}}$	g_8
$\bar{B}_s^0 \rightarrow K^0 \bar{Y}_{\pi u}$	$-\frac{f_8}{2}$	$\bar{B}^0 \rightarrow \bar{K}^0 \bar{Y}_{\pi u}$	$-\frac{h_8}{2}$
$\bar{B}_s^0 \rightarrow K^0 \bar{Y}_{\eta u}$	$\frac{f_8-2h_8}{2\sqrt{3}}$	$\bar{B}^0 \rightarrow \bar{K}^0 \bar{Y}_{\eta u}$	$\frac{h_8-2f_8}{2\sqrt{3}}$
$\bar{B}_s^0 \rightarrow \pi^- \bar{Y}_{\pi s}$	$\frac{h_8-g_8}{2}$	$\bar{B}^0 \rightarrow K^- \bar{Y}_{\pi d}$	$\frac{h_8}{2}$
$\bar{B}_s^0 \rightarrow \pi^- \bar{Y}_{\eta s}$	$\frac{g_8+h_8}{2\sqrt{3}}$	$\bar{B}^0 \rightarrow K^- \bar{Y}_{\eta d}$	$\frac{h_8-2g_8}{2\sqrt{3}}$

For the $\bar{\mathbf{15}}$ representation, the effective Hamiltonian is similar:

$$A_{B\bar{X}_c} = e_8 B_i H_j^i(\mathbf{8}) \bar{X}_k^{\{j,l\}} P_l^k + f_8 B_i H_j^k(\mathbf{8}) \bar{X}_k^{\{j,l\}} P_l^i + g_8 B_j H_i^k(\mathbf{8}) \bar{X}_k^{\{j,l\}} P_l^i + h_8 B_j H_l^i(\mathbf{8}) \bar{X}_k^{\{j,l\}} P_i^k. \quad (30)$$

Results for $A_{B\bar{X}_c}$ are obtained by expanding the above expressions and summarized in Tables III and IV.

C. $B \rightarrow X_c P$ decays

For the anti-charm production, the operator having the quark contents $(\bar{u}b)(\bar{q}c)$ is given by

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ub} V_{cq}^* [C_1 O_1^{\bar{u}c} + C_2 O_2^{\bar{u}c}] + \text{H.c.} \quad (31)$$

The two light anti-quarks form the $\mathbf{3}$ and $\bar{\mathbf{6}}$ representations. The anti-symmetric tensor $H(\mathbf{3})$ and the symmetric tensor $H(\bar{\mathbf{6}})$ have nonzero components

$$H^{13}(\mathbf{3}) = -H^{31}(\mathbf{3}) = 1, \quad H^{13}(\bar{\mathbf{6}}) = H^{31}(\bar{\mathbf{6}}) = 1, \quad (32)$$

TABLE V: Decay amplitudes of $B \rightarrow X_c P$ decays into the $\bar{\mathbf{6}}$.

channel $\Delta S = 0$	amplitude	channel $\Delta S = 1$	amplitude
$B^- \rightarrow K^+ X'_{ds\bar{u}}$	$\frac{a_6+b_3+b_6+c_3}{\sqrt{2}}$	$B^- \rightarrow \pi^+ X'_{ds\bar{u}}$	$-\frac{a_6+b_3+b_6+c_3}{\sqrt{2}}$
$B^- \rightarrow K^- X'_{ud\bar{s}}$	$\frac{2a_3-a_6-b_3}{\sqrt{2}}$	$B^- \rightarrow \pi^- X'_{sud}$	$\frac{-2a_3+a_6+b_3}{\sqrt{2}}$
$B^- \rightarrow \pi^- Y'_{(d\bar{d},s\bar{s})u}$	$a_3 - \frac{a_6}{2} - \frac{b_3}{2}$	$B^- \rightarrow K^- Y'_{(d\bar{d},s\bar{s})u}$	$\frac{-2a_3+a_6+b_3}{2}$
$B^- \rightarrow K^0 Y'_{(u\bar{u},d\bar{d})s}$	$\frac{a_6+b_3-b_6+c_3}{2}$	$B^- \rightarrow \bar{K}^0 Y'_{(u\bar{u},s\bar{s})d}$	$\frac{-a_6-b_3+b_6-c_3}{2}$
$B^- \rightarrow \pi^0 Y'_{(u\bar{u},s\bar{s})d}$	$\frac{2a_3-a_6-b_3-2b_6}{2\sqrt{2}}$	$B^- \rightarrow \pi^0 Y'_{(u\bar{u},d\bar{d})s}$	$\frac{-2a_3+2a_6+2b_3+b_6+c_3}{2\sqrt{2}}$
$B^- \rightarrow \eta Y'_{(u\bar{u},s\bar{s})d}$	$\frac{2a_3-3a_6-3b_3-2c_3}{2\sqrt{6}}$	$B^- \rightarrow \eta Y'_{(u\bar{u},d\bar{d})s}$	$-\frac{2a_3-3b_6+c_3}{2\sqrt{6}}$
$\bar{B}^0 \rightarrow K^0 X'_{s\bar{u}d}$	$\frac{-a_6+b_3-b_6+c_3}{\sqrt{2}}$	$\bar{B}_s^0 \rightarrow \bar{K}^0 X'_{ud\bar{s}}$	$\frac{a_6-b_3+b_6-c_3}{\sqrt{2}}$
$\bar{B}^0 \rightarrow \bar{K}^0 X'_{ud\bar{s}}$	$\frac{2a_3+a_6-b_3}{\sqrt{2}}$	$\bar{B}_s^0 \rightarrow K^0 X'_{s\bar{u}d}$	$\frac{-2a_3-a_6+b_3}{\sqrt{2}}$
$\bar{B}^0 \rightarrow \pi^+ Y'_{(u\bar{u},s\bar{s})d}$	$\frac{2a_3+a_6-b_3}{2}$	$\bar{B}_s^0 \rightarrow K^+ Y'_{(u\bar{u},d\bar{d})s}$	$\frac{-2a_3-a_6+b_3}{2}$
$\bar{B}^0 \rightarrow K^+ Y'_{(u\bar{u},d\bar{d})s}$	$\frac{-a_6+b_3+b_6+c_3}{2}$	$\bar{B}_s^0 \rightarrow \pi^+ Y'_{(u\bar{u},s\bar{s})d}$	$\frac{a_6-b_3-b_6-c_3}{2}$
$\bar{B}^0 \rightarrow \pi^0 Y'_{(d\bar{d},s\bar{s})u}$	$\frac{-2a_3+a_6-b_3+2b_6}{2\sqrt{2}}$	$\bar{B}_s^0 \rightarrow \pi^0 Y'_{(d\bar{d},s\bar{s})u}$	$\frac{-a_6+b_3+b_6+c_3}{2\sqrt{2}}$
$\bar{B}^0 \rightarrow \eta Y'_{(d\bar{d},s\bar{s})u}$	$\frac{2a_3+3a_6-3b_3-2c_3}{2\sqrt{6}}$	$\bar{B}_s^0 \rightarrow \eta Y'_{(d\bar{d},s\bar{s})u}$	$\frac{4a_3+3a_6-3b_3+3b_6-c_3}{2\sqrt{6}}$
$\bar{B}_s^0 \rightarrow K^0 Y'_{(d\bar{d},s\bar{s})u}$	$\frac{2a_3-b_6+c_3}{2}$	$\bar{B}^0 \rightarrow \bar{K}^0 Y'_{(d\bar{d},s\bar{s})u}$	$\frac{-2a_3+b_6-c_3}{2}$
$\bar{B}_s^0 \rightarrow K^+ Y'_{(u\bar{u},s\bar{s})d}$	$\frac{2a_3+b_6+c_3}{2}$	$\bar{B}^0 \rightarrow \pi^+ Y'_{(u\bar{u},d\bar{d})s}$	$\frac{-2a_3-b_6-c_3}{2}$
$\bar{B}_s^0 \rightarrow \pi^0 X'_{ud\bar{s}}$	$-b_6$	$\bar{B}^0 \rightarrow \pi^0 X'_{s\bar{u}d}$	$\frac{2a_3+b_6+c_3}{2}$
$\bar{B}_s^0 \rightarrow \eta X'_{ud\bar{s}}$	$-\frac{2a_3+c_3}{\sqrt{3}}$	$\bar{B}^0 \rightarrow \eta X'_{s\bar{u}d}$	$-\frac{2a_3-3b_6+c_3}{2\sqrt{3}}$

for the $b \rightarrow u\bar{c}s$ transition. For the transition $b \rightarrow u\bar{c}d$ one requests the interchange of $2 \leftrightarrow 3$ in the subscripts. The first kind of decays is proportional to $|V_{cb}V_{ud}^*| \sim \Lambda\lambda^2$ while the second of decays is proportional to $|V_{cd}V_{ub}^*| \sim \Lambda\lambda^4$, thus the latter is greatly suppressed compared to the first ones.

Charmed tetraquarks can be produced by the $b \rightarrow u\bar{c}d(s)$ transition. The irreducible $SU(3)$ amplitudes can be obtained by repeating previous procedure for $B \rightarrow \bar{X}_c P$ decays. We normalize $A(B \rightarrow X_c P) = V_{ub}V_{cq}^* A_{BX_c}$. For the $\bar{\mathbf{6}}$ charmed tetraquarks, the effective Hamiltonian can be constructed as

$$\begin{aligned}
A_{BX_c} = & a_3 B_i H^{[jl]}(\mathbf{3}) X_{[j,l]}^k P_k^i + b_3 B_i H^{[ij]}(\mathbf{3}) X_{[j,l]}^k P_k^l + c_3 B_k H^{[ij]}(\mathbf{3}) X_{[j,l]}^k P_i^l \\
& + a_6 B_i H^{\{ij\}}(\mathbf{6}) X_{[j,l]}^k P_k^l + b_6 B_k H^{\{ij\}}(\mathbf{6}) X_{[j,l]}^k P_i^l,
\end{aligned} \tag{33}$$

with the similar expression for the $\mathbf{15}$ representation:

$$\begin{aligned}
A_{BX_c} = & d_3 B_i H^{[ij]}(\mathbf{3}) X_{\{j,l\}}^k P_k^l + e_3 B_k H^{[ij]}(\mathbf{3}) X_{\{j,l\}}^k P_i^l \\
& + d_6 B_i H^{\{jl\}}(\mathbf{6}) X_{\{j,l\}}^k P_k^i + e_6 B_i H^{\{ij\}}(\mathbf{6}) X_{\{j,l\}}^k P_k^l + f_6 B_j H^{\{ij\}}(\mathbf{6}) X_{\{j,l\}}^k P_i^l.
\end{aligned} \tag{34}$$

The results for A_{BX_c} are collected in Tables V, and VI.

IV. RESULTS AND DISCUSSIONS: DECAY WIDTHS AND CP VIOLATION

Expanding the irreducible $SU(3)$ amplitudes obtained in the previous section, one obtains the decay amplitudes for each individual decays. They are listed in Tables I to VI. Due to the non-perturbative nature of QCD, we are far from being able to calculate these amplitudes from first principle. We will not attempt to do so in this work. Instead, we will use the amplitudes obtained to derive relations which may be testable experimentally. We find three types of relations of interest which related two different decays. We provide some details in the following.

A. Equal or proportional decay widths with known coefficients

There are decays with equal or proportional decay widths. This class of decays happen among $\Delta S = 0$ or $\Delta S = 1$ separately which can be extracted from Tables I to VI. There are several of them. For convenience, we summarize

TABLE VI: Decay amplitudes of $B \rightarrow X_c P$ decays into the **15** representation.

channel $\Delta S = 0$	amplitude	channel $\Delta S = 1$	amplitude
$B^- \rightarrow K^+ X_{ds\bar{u}}$	$\frac{d_3+e_3+e_6+f_6}{\sqrt{2}}$	$B^- \rightarrow \pi^+ X_{ds\bar{u}}$	$\frac{d_3+e_3+e_6+f_6}{\sqrt{2}}$
$B^- \rightarrow K^- X_{ud\bar{s}}$	$\frac{d_3+2d_6+e_6}{\sqrt{2}}$	$B^- \rightarrow \pi^- X_{su\bar{d}}$	$\frac{d_3+2d_6+e_6}{\sqrt{2}}$
$B^- \rightarrow \pi^+ Z_{dd\bar{u}}$	$d_3 + e_3 + e_6 + f_6$	$B^- \rightarrow K^+ Z_{ss\bar{u}}$	$d_3 + e_3 + e_6 + f_6$
$B^- \rightarrow \bar{K}^0 Z_{dd\bar{s}}$	$d_3 + e_6$	$B^- \rightarrow K^0 Z_{ss\bar{d}}$	$d_3 + e_6$
$B^- \rightarrow \pi^- Y_{\pi u}$	$\frac{-d_3-2d_6-\sqrt{2}e_3-e_6+\sqrt{2}f_6}{2}$	$B^- \rightarrow K^- Y_{\pi u}$	$\frac{f_6-e_3}{\sqrt{2}}$
$B^- \rightarrow \pi^- Y_{\eta u}$	$\frac{d_3+2d_6-\sqrt{2}e_3+e_6+\sqrt{2}f_6}{2\sqrt{3}}$	$B^- \rightarrow K^- Y_{\eta u}$	$-\frac{2d_3+4d_6+\sqrt{2}e_3+2e_6-\sqrt{2}f_6}{2\sqrt{3}}$
$B^- \rightarrow \pi^0 Y_{\pi d}$	$\frac{(2+\sqrt{2})d_3+2\sqrt{2}d_6+2\sqrt{2}e_3+\sqrt{2}e_6+2e_6}{4}$	$B^- \rightarrow \pi^0 Y_{\pi s}$	$\frac{2d_3+2d_6+e_3+2e_6+f_6}{2\sqrt{2}}$
$B^- \rightarrow \pi^0 Y_{\eta d}$	$\frac{(\sqrt{2}-2)d_3+2\sqrt{2}d_6+2\sqrt{2}e_3+\sqrt{2}e_6-2e_6}{4\sqrt{3}}$	$B^- \rightarrow \pi^0 Y_{\eta s}$	$\frac{2d_6+e_3+f_6}{2\sqrt{6}}$
$B^- \rightarrow K^0 Y_{\pi s}$	$\frac{-d_3-e_3-e_6+f_6}{2}$	$B^- \rightarrow \bar{K}^0 Y_{\pi d}$	$\frac{f_6-e_3}{2}$
$B^- \rightarrow K^0 Y_{\eta s}$	$\frac{d_3-e_3+e_6+f_6}{2\sqrt{3}}$	$B^- \rightarrow \bar{K}^0 Y_{\eta d}$	$-\frac{2d_3+e_3+2e_6-f_6}{2\sqrt{3}}$
$B^- \rightarrow \eta Y_{\pi d}$	$\frac{(\sqrt{2}-2)d_3+2\sqrt{2}d_6+\sqrt{2}e_6-2e_6+2\sqrt{2}f_6}{4\sqrt{3}}$	$B^- \rightarrow \eta Y_{\pi s}$	$\frac{2d_6+3e_3-f_6}{2\sqrt{6}}$
$B^- \rightarrow \eta Y_{\eta d}$	$\frac{(2+5\sqrt{2})d_3+2\sqrt{2}d_6+5\sqrt{2}e_6+2e_6+2\sqrt{2}f_6}{12}$	$B^- \rightarrow \eta Y_{\eta s}$	$\frac{6d_3+2d_6+3e_3+6e_6-f_6}{6\sqrt{2}}$
$\bar{B}^0 \rightarrow K^0 X_{su\bar{d}}$	$\frac{-d_3-e_3+e_6+f_6}{\sqrt{2}}$	$\bar{B}_s^0 \rightarrow \bar{K}^0 X_{ud\bar{s}}$	$\frac{-d_3-e_3+e_6+f_6}{\sqrt{2}}$
$\bar{B}^0 \rightarrow \bar{K}^0 X_{ud\bar{s}}$	$\frac{-d_3+2d_6+e_6}{\sqrt{2}}$	$\bar{B}_s^0 \rightarrow K^0 X_{su\bar{d}}$	$\frac{-d_3+2d_6+e_6}{\sqrt{2}}$
$\bar{B}^0 \rightarrow \pi^- Z_{uu\bar{d}}$	$-d_3 - e_3 + e_6 + f_6$	$\bar{B}_s^0 \rightarrow K^- Z_{uu\bar{s}}$	$-d_3 - e_3 + e_6 + f_6$
$\bar{B}^0 \rightarrow K^- Z_{uu\bar{s}}$	$e_6 - d_3$	$\bar{B}_s^0 \rightarrow \pi^- Z_{uu\bar{d}}$	$e_6 - d_3$
$\bar{B}^0 \rightarrow \pi^+ Y_{\pi d}$	$\frac{-d_3+2d_6-\sqrt{2}e_3+e_6-\sqrt{2}f_6}{2}$	$\bar{B}_s^0 \rightarrow K^+ Y_{\pi s}$	$\frac{-d_3+2d_6+e_6}{2}$
$\bar{B}^0 \rightarrow \pi^+ Y_{\eta d}$	$\frac{-d_3+2d_6+\sqrt{2}e_3+e_6+\sqrt{2}f_6}{2\sqrt{3}}$	$\bar{B}_s^0 \rightarrow K^+ Y_{\eta s}$	$-\frac{d_3-2d_6+2e_3-e_6+2f_6}{2\sqrt{3}}$
$\bar{B}^0 \rightarrow \pi^0 Y_{\pi u}$	$\frac{-(2+\sqrt{2})d_3+2\sqrt{2}d_6-2\sqrt{2}e_3+\sqrt{2}e_6+2e_6}{4}$	$\bar{B}_s^0 \rightarrow \pi^0 Y_{\pi u}$	$-\frac{(2+\sqrt{2})(d_3-e_6)}{4}$
$\bar{B}^0 \rightarrow \pi^0 Y_{\eta u}$	$\frac{(\sqrt{2}-2)d_3-2\sqrt{2}d_6+2\sqrt{2}e_3-\sqrt{2}e_6+2e_6}{4\sqrt{3}}$	$\bar{B}_s^0 \rightarrow \pi^0 Y_{\eta u}$	$\frac{(\sqrt{2}-2)d_3-2\sqrt{2}e_3-\sqrt{2}e_6+2e_6-2\sqrt{2}f_6}{4\sqrt{3}}$
$\bar{B}^0 \rightarrow K^+ Y_{\pi s}$	$\frac{-d_3-e_3+e_6-f_6}{2}$	$\bar{B}_s^0 \rightarrow \pi^+ Y_{\pi d}$	$\frac{e_6-d_3}{2}$
$\bar{B}^0 \rightarrow K^+ Y_{\eta s}$	$\frac{-d_3+e_3+e_6+f_6}{2\sqrt{3}}$	$\bar{B}_s^0 \rightarrow \pi^+ Y_{\eta d}$	$-\frac{d_3+2e_3-e_6+2f_6}{2\sqrt{3}}$
$\bar{B}^0 \rightarrow \eta Y_{\pi u}$	$\frac{(\sqrt{2}-2)d_3-2\sqrt{2}d_6-\sqrt{2}e_6+2e_6-2\sqrt{2}f_6}{4\sqrt{3}}$	$\bar{B}_s^0 \rightarrow \eta Y_{\pi u}$	$\frac{(\sqrt{2}-2)(d_3-e_6)}{4\sqrt{3}}$
$\bar{B}^0 \rightarrow \eta Y_{\eta u}$	$\frac{-(2+5\sqrt{2})d_3+2\sqrt{2}d_6+5\sqrt{2}e_6+2e_6+2\sqrt{2}f_6}{12}$	$\bar{B}_s^0 \rightarrow \eta Y_{\eta u}$	$\frac{-(2+5\sqrt{2})d_3+8\sqrt{2}d_6-6\sqrt{2}e_3+5\sqrt{2}e_6+2e_6+2\sqrt{2}f_6}{12}$
$\bar{B}_s^0 \rightarrow \pi^0 X_{ud\bar{s}}$	e_3	$\bar{B}^0 \rightarrow \pi^0 X_{su\bar{d}}$	$\frac{-2d_6+e_3+f_6}{2}$
$\bar{B}_s^0 \rightarrow \eta X_{ud\bar{s}}$	$\frac{f_6-2d_6}{\sqrt{3}}$	$\bar{B}^0 \rightarrow \eta X_{su\bar{d}}$	$\frac{2d_6+3e_3-f_6}{2\sqrt{3}}$
$\bar{B}_s^0 \rightarrow \pi^+ Z_{dd\bar{s}}$	$e_3 + f_6$	$\bar{B}^0 \rightarrow K^+ Z_{ss\bar{d}}$	$e_3 + f_6$
$\bar{B}_s^0 \rightarrow \pi^- Z_{uu\bar{s}}$	$f_6 - e_3$	$\bar{B}^0 \rightarrow K^- Z_{uu\bar{d}}$	$f_6 - e_3$
$\bar{B}_s^0 \rightarrow K^+ Y_{\pi d}$	d_6	$\bar{B}^0 \rightarrow \pi^+ Y_{\pi s}$	$d_6 - \frac{e_3}{2} - \frac{f_6}{2}$
$\bar{B}_s^0 \rightarrow K^+ Y_{\eta d}$	$\frac{d_6-e_3-f_6}{\sqrt{3}}$	$\bar{B}^0 \rightarrow \pi^+ Y_{\eta s}$	$\frac{2d_6+e_3+f_6}{2\sqrt{3}}$
$\bar{B}_s^0 \rightarrow K^0 Y_{\pi u}$	$-d_6$	$\bar{B}^0 \rightarrow \bar{K}^0 Y_{\pi u}$	$\frac{e_3-f_6}{2}$
$\bar{B}_s^0 \rightarrow K^0 Y_{\eta u}$	$\frac{d_6+e_3-f_6}{\sqrt{3}}$	$\bar{B}^0 \rightarrow \bar{K}^0 Y_{\eta u}$	$-\frac{4d_6+e_3-f_6}{2\sqrt{3}}$

them in Table VII. In Tables VII to X the left three columns are for the **15**-plet or $\bar{\mathbf{15}}$ -plet, while the right three columns are for the **6**-plet or $\bar{\mathbf{6}}$ -plet. In Table VII R is defined as the ratio of decay widths,

$$R = \Gamma_{channel-1}/\Gamma_{channel-2} \quad (35)$$

of decay modes for “channel 1” and “channel 2” on the same line. For the pairs of such decays, all have equal CP asymmetry $A_{CP}(channel-i)$. For example, for the first pairs on the left in the table, one has

$$\Gamma(B_c^- \rightarrow \pi^+ Z_{dd\bar{u}}) = 2\Gamma(B_c^- \rightarrow K^+ X_{ds\bar{u}}), \quad A_{CP}(B_c^- \rightarrow \pi^+ Z_{dd\bar{u}}) = A_{CP}(B_c^- \rightarrow K^+ X_{ds\bar{u}}). \quad (36)$$

Similarly, one can read from the table for other decay modes. The relations in Table VII hold if isospin symmetry

TABLE VII: SU(3) relations for decay widths for the $\mathbf{6}(\bar{\mathbf{6}})$ and $\mathbf{15}(\bar{\mathbf{15}})$. Herein and in the following tables, the left three columns are for the $\mathbf{15}$ or $\bar{\mathbf{15}}$; while the right three columns are for the $\mathbf{6}$ or $\bar{\mathbf{6}}$. R denotes the ratio of two decay widths.

channel 1	channel 2	R	channel 1	channel 2	R
$B_c^- \rightarrow \pi^+ Z_{dd\bar{u}}$	$B_c^- \rightarrow K^+ X_{ds\bar{u}}$	2	$B_c^- \rightarrow K^- X'_{ud\bar{s}}$	$B_c^- \rightarrow \pi^- Y'_{(d\bar{d},s\bar{s})u}$	2
$B_c^- \rightarrow K^+ Z_{ss\bar{u}}$	$B_c^- \rightarrow \pi^+ X_{ds\bar{u}}$	2	$B_c^- \rightarrow \pi^- X'_{sud}$	$B_c^- \rightarrow K^- Y'_{(d\bar{d},s\bar{s})u}$	2
$\bar{B}^0 \rightarrow \pi^+ \bar{Z}_{uud}$	$\bar{B}^0 \rightarrow \bar{K}^0 \bar{X}_{sud}$	2	$B^- \rightarrow K^- \bar{X}'_{sud}$	$B^- \rightarrow \pi^- \bar{Y}'_{(d\bar{d},s\bar{s})u}$	2
$\bar{B}^0 \rightarrow \pi^0 \bar{X}_{sud}$	$\bar{B}^0 \rightarrow \pi^- \bar{Y}_{\pi s}$	1	$\bar{B}^0 \rightarrow K^0 \bar{X}'_{ud\bar{s}}$	$\bar{B}^0 \rightarrow \pi^- \bar{Y}'_{(u\bar{u},s\bar{s})d}$	2
$\bar{B}^0 \rightarrow K^+ \bar{Z}_{uud}$	$\bar{B}^0 \rightarrow K^0 \bar{Y}_{\pi u}$	4	$\bar{B}^0 \rightarrow \pi^0 \bar{X}'_{sud}$	$\bar{B}^0 \rightarrow \pi^- \bar{Y}'_{(u\bar{u},d\bar{d})s}$	1
$B^- \rightarrow \pi^0 \bar{Z}_{uus}$	$B^- \rightarrow \pi^- \bar{X}_{ud\bar{s}}$	1	$B^- \rightarrow \pi^- \bar{X}'_{ud\bar{s}}$	$B^- \rightarrow K^- \bar{Y}'_{(d\bar{d},s\bar{s})u}$	2
$B^- \rightarrow \bar{K}^0 \bar{Z}_{uud}$	$B^- \rightarrow K^- \bar{Y}_{\pi u}$	2	$B^- \rightarrow \pi^- \bar{X}'_{ud\bar{s}}$	$\bar{B}^0 \rightarrow \pi^0 \bar{X}'_{ud\bar{s}}$	2
$B^- \rightarrow \bar{K}^0 \bar{Z}_{uud}$	$\bar{B}^0 \rightarrow \bar{K}^0 \bar{Y}_{\pi u}$	4	$\bar{B}^0 \rightarrow \pi^- \bar{Y}'_{(u\bar{u},s\bar{s})d}$	$\bar{B}^0 \rightarrow \pi^0 \bar{Y}'_{(d\bar{d},s\bar{s})u}$	2
$B^- \rightarrow \bar{K}^0 \bar{Z}_{uud}$	$\bar{B}^0 \rightarrow K^- \bar{Y}_{\pi d}$	4	$\bar{B}^0 \rightarrow \bar{K}^0 \bar{X}'_{sud}$	$\bar{B}^0 \rightarrow K^- \bar{Y}'_{(u\bar{u},d\bar{d})s}$	2
$B^- \rightarrow \eta \bar{Z}_{uus}$	$\bar{B}^0 \rightarrow \eta \bar{X}_{ud\bar{s}}$	2			
$\bar{B}^0 \rightarrow \pi^+ \bar{Z}_{uud}$	$\bar{B}^0 \rightarrow \pi^0 \bar{Y}_{\pi u}$	$8(3 - 2\sqrt{2})$			
$\bar{B}^0 \rightarrow \pi^+ \bar{Z}_{uud}$	$\bar{B}^0 \rightarrow \pi^- \bar{Y}_{\pi d}$	4			
$\bar{B}^0 \rightarrow \pi^- \bar{Y}_{\pi d}$	$\bar{B}^0 \rightarrow \eta \bar{Y}_{\pi u}$	$6(3 + 2\sqrt{2})$			
$\bar{B}^0 \rightarrow K^+ \bar{Z}_{uus}$	$\bar{B}^0 \rightarrow K^0 \bar{X}_{ud\bar{s}}$	2			
$\bar{B}^0 \rightarrow \bar{K}^0 \bar{X}_{sud}$	$\bar{B}^0 \rightarrow K^- \bar{Y}_{\pi s}$	2			
$B^- \rightarrow \pi^+ Z_{dd\bar{u}}$	$B^- \rightarrow K^+ X_{ds\bar{u}}$	2	$B^- \rightarrow K^- X'_{ud\bar{s}}$	$B^- \rightarrow \pi^- Y'_{(d\bar{d},s\bar{s})u}$	2
$\bar{B}^0 \rightarrow \pi^- Z_{uud}$	$\bar{B}^0 \rightarrow K^0 X_{sud}$	2	$\bar{B}^0 \rightarrow \bar{K}^0 X'_{ud\bar{s}}$	$\bar{B}^0 \rightarrow \pi^+ Y'_{(u\bar{u},s\bar{s})d}$	2
$\bar{B}^0 \rightarrow K^+ Y_{\pi d}$	$\bar{B}^0 \rightarrow K^0 Y_{\pi u}$	1			
$B^- \rightarrow K^+ Z_{ss\bar{u}}$	$B^- \rightarrow \pi^+ X_{ds\bar{u}}$	2	$B^- \rightarrow \pi^- X'_{sud}$	$B^- \rightarrow K^- Y'_{(d\bar{d},s\bar{s})u}$	2
$B^- \rightarrow \bar{K}^0 Y_{\pi d}$	$\bar{B}^0 \rightarrow \bar{K}^0 Y_{\pi u}$	1	$\bar{B}^0 \rightarrow \pi^+ Y'_{(u\bar{u},d\bar{d})s}$	$\bar{B}^0 \rightarrow \pi^0 X'_{sud}$	1
$B^- \rightarrow K^- Y_{\pi u}$	$B^- \rightarrow \bar{K}^0 Y_{\pi d}$	2	$\bar{B}^0 \rightarrow \eta X'_{sud}$	$B^- \rightarrow \eta Y'_{(u\bar{u},d\bar{d})s}$	2
$\bar{B}^0 \rightarrow \pi^+ Y_{\pi s}$	$\bar{B}^0 \rightarrow \pi^0 X_{sud}$	1	$\bar{B}^0 \rightarrow \pi^+ Y'_{(u\bar{u},s\bar{s})d}$	$\bar{B}^0 \rightarrow \pi^0 Y'_{(d\bar{d},s\bar{s})u}$	2
$\bar{B}^0 \rightarrow \pi^+ Y_{\eta s}$	$B^- \rightarrow \pi^0 Y_{\eta s}$	2	$\bar{B}^0 \rightarrow K^0 X'_{sud}$	$\bar{B}^0 \rightarrow K^+ Y'_{(u\bar{u},d\bar{d})s}$	2
$\bar{B}^0 \rightarrow K^- Z_{uud}$	$B^- \rightarrow \bar{K}^0 Y_{\pi d}$	4			
$\bar{B}^0 \rightarrow \eta X_{sud}$	$B^- \rightarrow \eta Y_{\pi s}$	2			
$\bar{B}^0 \rightarrow \pi^+ Y_{\pi d}$	$\bar{B}^0 \rightarrow \eta Y_{\pi u}$	$6(3 + 2\sqrt{2})$			
$\bar{B}^0 \rightarrow \pi^- Z_{uud}$	$\bar{B}^0 \rightarrow \pi^+ Y_{\pi d}$	4			
$\bar{B}^0 \rightarrow \pi^- Z_{uud}$	$\bar{B}^0 \rightarrow \pi^0 Y_{\pi u}$	$8(3 - 2\sqrt{2})$			
$\bar{B}^0 \rightarrow K^0 X_{sud}$	$\bar{B}^0 \rightarrow K^+ Y_{\pi s}$	2			
$\bar{B}^0 \rightarrow K^- Z_{uus}$	$\bar{B}^0 \rightarrow \bar{K}^0 X_{ud\bar{s}}$	2			

holds. They are broken by small isospin violating effects. Measurements of the above equalities can provide details information about charmed tetraquarks.

We comment that CP asymmetry A_{CP} in $B_c \rightarrow X_c P$ can be sizeable, of order 10% similar to $B \rightarrow PP$, due to both tree and penguin contributions. While for CP asymmetry A_{CP} for $B \rightarrow \bar{X}_c P$ and $B \rightarrow X_c P$ are much smaller since there are only tree contributions.

B. U -spin relations for $B \rightarrow \bar{X}_c P$ and $B \rightarrow X_c P$ decays

In $B \rightarrow \bar{X}_c P$ and $B \rightarrow X_c P$ decays, several decay amplitudes are related by U -spin symmetry, the exchange of d and s quarks. This leads to the ratios of decay widths proportional to CKM matrix elements ratio $|V_{is}|^2/|V_{id}|^2$ with

TABLE VIII: U -spin relations for $B \rightarrow \bar{X}_c P$ decays involving both the **6** and **$\bar{15}$** . Results in the “channel 1” are for $b \rightarrow d$ processes and the ones in the “channel 2” are for $b \rightarrow s$ processes. r denotes the ratio of the two amplitudes.

channel 1	channel 2	r	channel 1	channel 2	r
$B^- \rightarrow K^0 \bar{Z}_{uu\bar{s}}$	$B^- \rightarrow \bar{K}^0 \bar{Z}_{uu\bar{d}}$	1	$B^- \rightarrow \pi^- \bar{Y}'_{(d\bar{d},s\bar{s})u}$	$B^- \rightarrow \pi^- \bar{X}'_{ud\bar{s}}$	$\frac{1}{\sqrt{2}}$
$B^- \rightarrow K^0 \bar{Z}_{uu\bar{s}}$	$B^- \rightarrow K^- \bar{Y}_{\pi u}$	$\sqrt{2}$	$B^- \rightarrow \pi^- \bar{Y}'_{(d\bar{d},s\bar{s})u}$	$B^- \rightarrow K^- \bar{Y}'_{(d\bar{d},s\bar{s})u}$	-1
$B^- \rightarrow K^0 \bar{Z}_{uu\bar{s}}$	$\bar{B}^0 \rightarrow \bar{K}^0 \bar{Y}_{\pi u}$	-2	$B^- \rightarrow \pi^- \bar{Y}'_{(d\bar{d},s\bar{s})u}$	$\bar{B}^0 \rightarrow \pi^0 \bar{X}'_{ud\bar{s}}$	-1
$B^- \rightarrow K^0 \bar{Z}_{uu\bar{s}}$	$\bar{B}^0 \rightarrow K^- \bar{Y}_{\pi d}$	2	$B^- \rightarrow K^- \bar{X}'_{su\bar{d}}$	$B^- \rightarrow \pi^- \bar{X}'_{ud\bar{s}}$	-1
$B^- \rightarrow K^- \bar{X}_{su\bar{d}}$	$B^- \rightarrow \pi^0 \bar{Z}_{uu\bar{s}}$	1	$B^- \rightarrow K^- \bar{X}'_{su\bar{d}}$	$B^- \rightarrow K^- \bar{Y}'_{(d\bar{d},s\bar{s})u}$	$\sqrt{2}$
$B^- \rightarrow K^- \bar{X}_{su\bar{d}}$	$B^- \rightarrow \pi^- \bar{X}_{ud\bar{s}}$	1	$B^- \rightarrow K^- \bar{X}'_{su\bar{d}}$	$\bar{B}^0 \rightarrow \pi^0 \bar{X}'_{ud\bar{s}}$	$\sqrt{2}$
$\bar{B}^0 \rightarrow \pi^+ \bar{Z}_{uu\bar{d}}$	$\bar{B}^0 \rightarrow K^+ \bar{Z}_{uu\bar{s}}$	1	$\bar{B}^0 \rightarrow \pi^- \bar{Y}'_{(u\bar{u},s\bar{s})d}$	$\bar{B}^0 \rightarrow \bar{K}^0 \bar{X}'_{su\bar{d}}$	$-\frac{1}{\sqrt{2}}$
$\bar{B}^0 \rightarrow \pi^+ \bar{Z}_{uu\bar{d}}$	$\bar{B}_s^0 \rightarrow K^0 \bar{X}_{ud\bar{s}}$	$\sqrt{2}$	$\bar{B}^0 \rightarrow \pi^- \bar{Y}'_{(u\bar{u},s\bar{s})d}$	$\bar{B}_s^0 \rightarrow K^- \bar{Y}'_{(u\bar{u},d\bar{d})s}$	-1
$\bar{B}^0 \rightarrow K^+ \bar{Z}_{uu\bar{s}}$	$\bar{B}_s^0 \rightarrow \pi^+ \bar{Z}_{uu\bar{d}}$	1	$\bar{B}^0 \rightarrow K^0 \bar{X}'_{ud\bar{s}}$	$\bar{B}_s^0 \rightarrow \bar{K}^0 \bar{X}'_{su\bar{d}}$	-1
$\bar{B}^0 \rightarrow K^+ \bar{Z}_{uu\bar{s}}$	$\bar{B}_s^0 \rightarrow \pi^0 \bar{Y}_{\pi u}$	$4 - 2\sqrt{2}$	$\bar{B}^0 \rightarrow K^0 \bar{X}'_{ud\bar{s}}$	$\bar{B}_s^0 \rightarrow K^- \bar{Y}'_{(u\bar{u},d\bar{d})s}$	$-\sqrt{2}$
$\bar{B}^0 \rightarrow K^+ \bar{Z}_{uu\bar{s}}$	$\bar{B}_s^0 \rightarrow \pi^- \bar{Y}_{\pi d}$	2	$\bar{B}^0 \rightarrow \bar{K}^0 \bar{X}'_{su\bar{d}}$	$\bar{B}_s^0 \rightarrow K^0 \bar{X}'_{ud\bar{s}}$	-1
$\bar{B}^0 \rightarrow K^+ \bar{Z}_{uu\bar{s}}$	$\bar{B}_s^0 \rightarrow \eta \bar{Y}_{\pi u}$	$-\frac{4\sqrt{3}}{\sqrt{2}-2}$	$\bar{B}^0 \rightarrow K^- \bar{Y}'_{(u\bar{u},d\bar{d})s}$	$\bar{B}_s^0 \rightarrow \pi^0 \bar{Y}'_{(d\bar{d},s\bar{s})u}$	$\sqrt{2}$
$\bar{B}^0 \rightarrow K^0 \bar{X}_{ud\bar{s}}$	$\bar{B}_s^0 \rightarrow \bar{K}^0 \bar{X}_{su\bar{d}}$	1	$\bar{B}^0 \rightarrow K^- \bar{Y}'_{(u\bar{u},d\bar{d})s}$	$\bar{B}_s^0 \rightarrow \pi^- \bar{Y}'_{(u\bar{u},s\bar{s})d}$	-1
$\bar{B}^0 \rightarrow K^0 \bar{X}_{ud\bar{s}}$	$\bar{B}_s^0 \rightarrow K^- \bar{Y}_{\pi s}$	$\sqrt{2}$	$\bar{B}_s^0 \rightarrow \pi^0 \bar{X}'_{su\bar{d}}$	$\bar{B}^0 \rightarrow K^- \bar{Y}'_{(u\bar{u},s\bar{s})d}$	1
$\bar{B}^0 \rightarrow \bar{K}^0 \bar{X}_{su\bar{d}}$	$\bar{B}_s^0 \rightarrow K^+ \bar{Z}_{uu\bar{s}}$	$\frac{1}{\sqrt{2}}$	$\bar{B}_s^0 \rightarrow \pi^- \bar{Y}'_{(u\bar{u},d\bar{d})s}$	$\bar{B}^0 \rightarrow K^- \bar{Y}'_{(u\bar{u},s\bar{s})d}$	-1
$\bar{B}^0 \rightarrow \bar{K}^0 \bar{X}_{su\bar{d}}$	$\bar{B}_s^0 \rightarrow K^0 \bar{X}_{ud\bar{s}}$	1	$\bar{B}_s^0 \rightarrow K^0 \bar{Y}'_{(d\bar{d},s\bar{s})u}$	$\bar{B}^0 \rightarrow \bar{K}^0 \bar{Y}'_{(d\bar{d},s\bar{s})u}$	-1
$\bar{B}_s^0 \rightarrow K^+ \bar{Z}_{uu\bar{d}}$	$\bar{B}^0 \rightarrow \pi^+ \bar{Z}_{uu\bar{s}}$	1			
$\bar{B}_s^0 \rightarrow K^0 \bar{Y}_{\pi u}$	$\bar{B}^0 \rightarrow \pi^+ \bar{Z}_{uu\bar{s}}$	$-\frac{1}{2}$			
$\bar{B}_s^0 \rightarrow K^- \bar{Z}_{ss\bar{d}}$	$\bar{B}^0 \rightarrow \pi^- \bar{Z}_{dd\bar{s}}$	1			

$i = u, c$.

For $B \rightarrow \bar{X}_c P$ the decay amplitudes are equal to the irreducible amplitudes multiply the CKM factor $V_{cb}V_{ud}^*$ and $V_{cb}V_{us}^*$ for $\Delta S = 0$ and $\Delta S = 1$ decay modes, respectively. Therefore the pairs listed in Table VIII, the ratio of the decay widths will be given by

$$\frac{\Gamma(\text{channel} - 1)}{\Gamma(\text{channel} - 2)} = r^2 \frac{|V_{ud}|^2}{|V_{us}|^2}, \quad (37)$$

where the parameter r is defined by $r = A_{B\bar{X}_c}(\text{channel} - 1)/A_{B\bar{X}_c}(\text{channel} - 2)$.

For $B \rightarrow X_c P$ the decay amplitudes are equal to the irreducible amplitudes multiply the CKM factor $V_{ub}V_{cd}^*$ and $V_{ub}V_{cs}^*$ for $\Delta S = 0$ and $\Delta S = 1$ decay modes, respectively. Thereby for the pairs listed in Table VIII, the ratio of the decay widths will be given by

$$\frac{\Gamma(\text{channel} - 1)}{\Gamma(\text{channel} - 2)} = r^2 \frac{|V_{cd}|^2}{|V_{cs}|^2}. \quad (38)$$

In this case the parameter r is defined by $r = A_{BX_c}(\text{channel} - 1)/A_{BX_c}(\text{channel} - 2)$. We give the coefficient r for $B \rightarrow \bar{X}_c P$ and $B \rightarrow X_c P$ in Tables VIII and IX. One can read off the relations for pairs related by U -spin and test the symmetry by comparing the relations with data in future.

C. U -spin for $B_c \rightarrow X_c P$ decay and CP violating relations

For $B_c \rightarrow X_c P$ decays, there are two terms with different CKM factors. Although the U -spin symmetry can relate $\Delta S = 0$ and $\Delta S = 1$ decays, there is no simple rate relation as in the case for $B \rightarrow \bar{X}_c P$ and $B \rightarrow X_c P$ decays. However, there exists a relation for the CP violating quantity $\Delta = \Gamma - \bar{\Gamma}$. We now derive such a relation. Let us consider two U -spin connected decays with proportional amplitudes $A_{B_c}^T$ and $A_{B_c}^P$, with the decay amplitudes

$$\begin{aligned} A(\Delta S = 0) &= r (V_{ub}V_{ud}^* A_{B_c}^T + V_{tb}V_{td}^* A_{B_c}^P), \\ A(\Delta S = 1) &= V_{ub}V_{us}^* A_{B_c}^T + V_{tb}V_{ts}^* A_{B_c}^P. \end{aligned} \quad (39)$$

TABLE IX: U -spin relations for B decays involving both the $\bar{\mathbf{6}}$ and $\mathbf{15}$. Results in the “channel 1” are for $b \rightarrow d$ processes and the ones in the “channel 2” are for $b \rightarrow s$ processes. r denotes the ratio of the two amplitudes.

channel 1	channel 2	r	channel 1	channel 2	r
$B^- \rightarrow \pi^+ Z_{dd\bar{u}}$	$B^- \rightarrow \pi^+ X_{ds\bar{u}}$	$\sqrt{2}$	$B^- \rightarrow \pi^- Y'_{(d\bar{d},s\bar{s})u}$	$B^- \rightarrow \pi^- X'_{su\bar{d}}$	$-\frac{1}{\sqrt{2}}$
$B^- \rightarrow \pi^+ Z_{dd\bar{u}}$	$B^- \rightarrow K^+ Z_{ss\bar{u}}$	1	$B^- \rightarrow \pi^- Y'_{(d\bar{d},s\bar{s})u}$	$B^- \rightarrow K^- Y'_{(d\bar{d},s\bar{s})u}$	-1
$B^- \rightarrow K^+ X_{ds\bar{u}}$	$B^- \rightarrow \pi^+ X_{ds\bar{u}}$	1	$B^- \rightarrow K^+ X'_{ds\bar{u}}$	$B^- \rightarrow \pi^+ X'_{ds\bar{u}}$	-1
$B^- \rightarrow K^+ X_{ds\bar{u}}$	$B^- \rightarrow K^+ Z_{ss\bar{u}}$	$\frac{1}{\sqrt{2}}$	$B^- \rightarrow K^0 Y'_{(u\bar{u},d\bar{d})s}$	$B^- \rightarrow \bar{K}^0 Y'_{(u\bar{u},s\bar{s})d}$	-1
$B^- \rightarrow \bar{K}^0 Z_{dd\bar{s}}$	$B^- \rightarrow K^0 Z_{ss\bar{d}}$	1	$B^- \rightarrow K^- X'_{ud\bar{s}}$	$B^- \rightarrow \pi^- X'_{su\bar{d}}$	-1
$B^- \rightarrow K^- X_{ud\bar{s}}$	$B^- \rightarrow \pi^- X_{su\bar{d}}$	1	$B^- \rightarrow K^- X'_{ud\bar{s}}$	$B^- \rightarrow K^- Y'_{(d\bar{d},s\bar{s})u}$	$-\sqrt{2}$
$\bar{B}^0 \rightarrow \pi^- Z_{uu\bar{d}}$	$\bar{B}^0 \rightarrow \bar{K}^0 X_{ud\bar{s}}$	$\sqrt{2}$	$\bar{B}^0 \rightarrow \pi^+ Y'_{(u\bar{u},s\bar{s})d}$	$\bar{B}^0 \rightarrow K^+ Y'_{(u\bar{u},d\bar{d})s}$	-1
$\bar{B}^0 \rightarrow \pi^- Z_{uu\bar{d}}$	$\bar{B}^0 \rightarrow K^- Z_{uu\bar{s}}$	1	$\bar{B}^0 \rightarrow \pi^+ Y'_{(u\bar{u},s\bar{s})d}$	$\bar{B}^0 \rightarrow K^0 X'_{su\bar{d}}$	$-\frac{1}{\sqrt{2}}$
$\bar{B}^0 \rightarrow K^0 X_{su\bar{d}}$	$\bar{B}^0 \rightarrow \bar{K}^0 X_{ud\bar{s}}$	1	$\bar{B}^0 \rightarrow K^+ Y'_{(u\bar{u},d\bar{d})s}$	$\bar{B}^0 \rightarrow \pi^+ Y'_{(u\bar{u},s\bar{s})d}$	-1
$\bar{B}^0 \rightarrow K^0 X_{su\bar{d}}$	$\bar{B}^0 \rightarrow K^- Z_{uu\bar{s}}$	$\frac{1}{\sqrt{2}}$	$\bar{B}^0 \rightarrow K^+ Y'_{(u\bar{u},d\bar{d})s}$	$\bar{B}^0 \rightarrow \pi^0 Y'_{(d\bar{d},s\bar{s})u}$	$\sqrt{2}$
$\bar{B}^0 \rightarrow \bar{K}^0 X_{ud\bar{s}}$	$\bar{B}^0 \rightarrow K^+ Y_{\pi d}$	$\sqrt{2}$	$\bar{B}^0 \rightarrow K^0 X'_{su\bar{d}}$	$\bar{B}^0 \rightarrow \bar{K}^0 X'_{ud\bar{s}}$	-1
$\bar{B}^0 \rightarrow \bar{K}^0 X_{ud\bar{s}}$	$\bar{B}^0 \rightarrow K^0 X_{su\bar{d}}$	1	$\bar{B}^0 \rightarrow \bar{K}^0 X'_{ud\bar{s}}$	$\bar{B}^0 \rightarrow K^+ Y'_{(u\bar{u},d\bar{d})s}$	$-\sqrt{2}$
$\bar{B}^0 \rightarrow K^- Z_{uu\bar{s}}$	$\bar{B}^0 \rightarrow \pi^+ Y_{\pi d}$	2	$\bar{B}^0 \rightarrow \bar{K}^0 X'_{ud\bar{s}}$	$\bar{B}^0 \rightarrow K^0 X'_{su\bar{d}}$	-1
$\bar{B}^0 \rightarrow K^- Z_{uu\bar{s}}$	$\bar{B}^0 \rightarrow \pi^0 Y_{\pi u}$	$4 - 2\sqrt{2}$	$\bar{B}^0 \rightarrow K^+ Y'_{(u\bar{u},s\bar{s})d}$	$\bar{B}^0 \rightarrow \pi^+ Y'_{(u\bar{u},d\bar{d})s}$	-1
$\bar{B}^0 \rightarrow K^- Z_{uu\bar{s}}$	$\bar{B}^0 \rightarrow \pi^- Z_{uu\bar{d}}$	1	$\bar{B}^0 \rightarrow K^+ Y'_{(u\bar{u},s\bar{s})d}$	$\bar{B}^0 \rightarrow \pi^0 X'_{su\bar{d}}$	1
$\bar{B}^0 \rightarrow K^- Z_{uu\bar{s}}$	$\bar{B}^0 \rightarrow \eta Y_{\pi u}$	$-\frac{4\sqrt{3}}{\sqrt{2}-2}$	$\bar{B}^0 \rightarrow K^0 Y'_{(d\bar{d},s\bar{s})u}$	$\bar{B}^0 \rightarrow \bar{K}^0 Y'_{(d\bar{d},s\bar{s})u}$	-1
$\bar{B}_s^0 \rightarrow \pi^+ Z_{dd\bar{s}}$	$\bar{B}_s^0 \rightarrow K^+ Z_{ss\bar{d}}$	1			
$\bar{B}_s^0 \rightarrow \pi^- Z_{uu\bar{s}}$	$B^- \rightarrow \bar{K}^0 Y_{\pi d}$	2			
$\bar{B}_s^0 \rightarrow \pi^- Z_{uu\bar{s}}$	$B^- \rightarrow K^- Y_{\pi u}$	$\sqrt{2}$			
$\bar{B}_s^0 \rightarrow \pi^- Z_{uu\bar{s}}$	$\bar{B}^0 \rightarrow \bar{K}^0 Y_{\pi u}$	-2			
$\bar{B}_s^0 \rightarrow \pi^- Z_{uu\bar{s}}$	$\bar{B}^0 \rightarrow K^- Z_{uu\bar{d}}$	1			

TABLE X: U -spin relations for B_c decays involving both the $\bar{\mathbf{6}}$ -plet and $\mathbf{15}$ -plet. Results in the “channel 1” are for $b \rightarrow d$ processes and the ones in the “channel 2” are for $b \rightarrow s$ processes. r denotes the ratio of the two amplitudes.

channel 1	channel 2	r	channel 1	channel 2	r
$B_c^- \rightarrow \pi^+ Z_{dd\bar{u}}$	$B_c^- \rightarrow \pi^+ X_{ds\bar{u}}$	$\sqrt{2}$	$B_c^- \rightarrow \pi^- Y'_{(d\bar{d},s\bar{s})u}$	$B_c^- \rightarrow \pi^- X'_{su\bar{d}}$	$-\frac{1}{\sqrt{2}}$
$B_c^- \rightarrow \pi^+ Z_{dd\bar{u}}$	$B_c^- \rightarrow K^+ Z_{ss\bar{u}}$	1	$B_c^- \rightarrow \pi^- Y'_{(d\bar{d},s\bar{s})u}$	$B_c^- \rightarrow K^- Y'_{(d\bar{d},s\bar{s})u}$	-1
$B_c^- \rightarrow K^+ X_{ds\bar{u}}$	$B_c^- \rightarrow \pi^+ X_{ds\bar{u}}$	1	$B_c^- \rightarrow K^+ X'_{ds\bar{u}}$	$B_c^- \rightarrow \pi^+ X'_{ds\bar{u}}$	-1
$B_c^- \rightarrow K^+ X_{ds\bar{u}}$	$B_c^- \rightarrow K^+ Z_{ss\bar{u}}$	$\frac{1}{\sqrt{2}}$	$B_c^- \rightarrow K^0 Y'_{(u\bar{u},d\bar{d})s}$	$B_c^- \rightarrow \bar{K}^0 Y'_{(u\bar{u},s\bar{s})d}$	-1
$B_c^- \rightarrow \bar{K}^0 Z_{dd\bar{s}}$	$B_c^- \rightarrow K^0 Z_{ss\bar{d}}$	1	$B_c^- \rightarrow K^- X'_{ud\bar{s}}$	$B_c^- \rightarrow \pi^- X'_{su\bar{d}}$	-1
$B_c^- \rightarrow K^- X_{ud\bar{s}}$	$B_c^- \rightarrow \pi^- X_{su\bar{d}}$	1	$B_c^- \rightarrow K^- X'_{ud\bar{s}}$	$B_c^- \rightarrow K^- Y'_{(d\bar{d},s\bar{s})u}$	$-\sqrt{2}$

Using the Jarlskog relation $Im(V_{ub}V_{ud}^*V_{tb}^*V_{td}) = -Im(V_{ub}V_{us}^*V_{tb}^*V_{ts})$, the CP violating rate difference $\Delta(\Delta S = i) = \Gamma(\Delta S = i) - \bar{\Gamma}(\Delta S = i)$ will be related with [58–60]

$$r^2 \Delta(\Delta S = 0) = -\Delta(\Delta S = 1). \quad (40)$$

This leads to branching ratio $\mathcal{B}(\Delta S = i)$ and CP asymmetry $A_{CP}(\Delta S = i)$ relation,

$$\frac{A_{CP}(\Delta S = 0)}{A_{CP}(\Delta S = 1)} = -r^2 \frac{\mathcal{B}(\Delta S = 1)}{\mathcal{B}(\Delta S = 0)}. \quad (41)$$

In Table X, we collect the B_c decay pairs related by U -spin. CP asymmetries for these pairs satisfy relation in Eq.(41). As already mentioned, CP asymmetries for $B_c \rightarrow X_c P$ decays are expected to be similar to $B \rightarrow PP$ decays which can be of order 10%. Experimental measurements of these relations are important to test flavor $SU(3)$ symmetry and also CKM mechanism for CP violation.

V. CONCLUSIONS

In this work we have studied the charmed tetraquarks with three light flavors in weak decays of B_c and B mesons. If indeed a charmed tetraquark X_c with three different light quarks is discovered, they should come in a $\bar{\mathbf{6}}$ or a $\mathbf{15}$ multiplet of flavor $SU(3)$. A most direct consequence of the flavor $SU(3)$ symmetry is that tetraquarks are formed in irreducible representations. Therefore to test whether flavor $SU(3)$ symmetry is playing a role in organizing hadron states, one needs to find all of the states in a given multiplet. We have studied production of X_c in B_c and B weak decays. The total number of states with similar masses will be able to distinguish whether X_c is in $\bar{\mathbf{6}}$ or $\mathbf{15}$. The doubly charged states $Z_{dd\bar{u}}$ and $Z_{ss\bar{u}}$ are the smoking guns for X_c belonging to $\mathbf{15}$. We find a number of relations among decay branching ratio for $\Delta S = 0$ and $\Delta S = 1$ processes separately. We also find relations among the CP asymmetries and branching ratios for decays related by the U -spin symmetry. All these can serve to confirm the existence of tetraquark states and to study their properties. With more data from the LHCb and Belle-II, one may discover the X_c states and study their properties. We urge our experimental colleagues to carry out related analysis to learn more about hadron states built from multiquarks.

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